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BY

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PREFACE

Figures are simply tools that are a help in securing accuracy, in saving time and making a man more valuable to himself and others. True, many good mechanics get along with only enough mathematical knowledge to count up their wages, but the men who get to the top are not those who depend on others to tell them what gears to use, or the depth of a 9-pitch thread.

While the explanations in this book may be considered too elementary by some, I have simply tried to make every point so clear that any one could understand it, and to show how the methods apply to every-day shop work. I have given only such rules and calculations as have proved useful in the shop, and have omitted much that most schools consider necessary, because in my experience it only tends to confuse and is rarely used in actual work in the shop. And in each case I have tried to show the "why" of each step taken, so as to make a man as independent as possible of having to remember rules.

I shall appreciate suggestions and questions from any reader at any time.

The Author.

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CHAPTER I

COMMON FRACTIONS

If we could measure everything in even inches and never had to divide anything so as to make parts of an inch come in the answer, there would be very little use for fractions in the shop. But as this is not the case we must get used to odd dimensions and know how to figure them.

Suppose we have a piece of steel 9 inches long to be cut in four equal parts. If it was 8 inches, each part would be just 2 inches, but there is an inch left over, so we divide this in four parts also and get $\frac{1}{4}$ inch to add to the 2 inches, making $2\frac{1}{4}$ inches for each piece. You can try this with a rule, but it is well to depend on figures, as it isn't always convenient to measure things off with a rule. Dividing 9 by 4 gives

$$\begin{array}{r} 4)9(2 \\ \underline{8} \\ 1 \end{array}$$

and the number left over goes over the number we divide by, making it $\frac{1}{4}$, as we found before.

The number top of the line is called the numerator; that below the line the denominator; the number we divide by is the divisor; the number we divide it into is the dividend; the answer is the quotient, and the number left over is the remainder. In this example the parts are as follows:

$$\frac{1 \text{ (Numerator)}}{4 \text{ (Denominator)}} \text{ of } 9 = \begin{array}{lll} \text{Divisor} & \text{Dividend} & \text{Quotient} \\ 4) & 9 & (2\tfrac{1}{4} \\ & \underline{8} & \\ & 1 \text{ (Remainder)} & \end{array}$$

The quotient or answer is a mixed number, in which 2 is the whole number and $\frac{1}{4}$ the fraction.

To divide $3\frac{3}{16}$ into three parts it doesn't take any figuring to see that $\frac{1}{3}$ of 3 equals 1, and that $\frac{1}{3}$ of $\frac{3}{16}$ equals $\frac{1}{16}$ so that $\frac{1}{3}$ of $3\frac{3}{16}$ equals $1\frac{1}{16}$. Now suppose we have two holes in a piece of work, $4\frac{1}{8}$ inches apart, and want to put two holes in between them at equal distances. This means dividing the distance into three equal parts, but it doesn't divide as easily as the others in the way we have been doing. Going back to the first example, it is easy to see that we might have put the 9 over the 4, and made $\frac{9}{4}$, in which 9 is the numerator and 4 the denominator. To reduce this to a mixed number, $2\frac{1}{4}$, we divide 9 by 4, so we see the numerator is the same as the dividend and the denominator the same as the divisor. In other words, $\frac{9}{4}$ is the same as $2\frac{1}{4}$, only expressed in a different way, and in any fraction the number above the line is to be divided by the number below it. When the numerator is larger than the denominator it is called an improper fraction, because it can be reduced to a mixed number.

It is often easier to divide an improper fraction than a mixed number, and $4\frac{1}{8}$ to be divided into thirds is an example of this. So we change it back to a mixed number by multiplying the whole number by the denominator of the fraction, and adding the numerator to it. This gives 8 times 4 equals 32, plus 1 equals 33, and don't forget that the 8 must go underneath, as it is $\frac{33}{8}$ and not 33 whole

ones. Dividing 33/8 by 3 gives 11/8, so we know that 11/8 is exactly one third of 33/8 or 4 1/8, and reducing 11/8 to a mixed number by dividing 11 by 8 we have 1 and 3 eighths left over, so one third of 4 1/8 is 1 3/8, and it is easier to do it in this way.

Different Ways of Expressing the Same Value

A curious thing about fractions is that you can express the same thing in such a variety of ways; as 1/2 and 2/4 and 3/6 and 5/10 and 7/14 and 16/32 all have the same value, because the denominator is just double the numerator in every case. You can multiply a fraction by a whole number by either multiplying the numerator or dividing the denominator by that number. Suppose you wish to multiply 1/4 inch by 2. You could either multiply the numerator and get 2/4, or divide the denominator and get 1/2, both being of the same value. The last way is the easiest, when it comes out even, but to multiply 1/3 by 2 it is easier to use the first method and multiply the numerator, making it 2/3.

The reverse is true when we come to divide fractions, as to divide 2/3 by 2 we either divide the numerator and get 1/3, or multiply the denominator and get 2/6 which is exactly the same thing. It is easier to divide the numerator when it can be done evenly, as that gives the fraction in lower terms and is to be preferred. By lower terms is meant smaller numbers in both numerator and denominator. Both 10/30 and 1/3 have the same value, and in the case of large numbers see how many times they can be divided by the same number. If you have 17/32 it is at its lowest terms, because there is no number which will divide them both evenly, but with 18/32 we can divide both by 2 and get 9/16 as

the lowest, or with $\frac{16}{32}$ we can divide both terms four times by 2, or divide both by 16 and get $\frac{1}{2}$ in either case.

It often happens that we have to multiply fractions by whole numbers and divide fractions by fractions, but this is not hard if we go at it understandingly. If we lay three test blocks each $\frac{3}{8}$ of an inch side by side, we have practically multiplied $\frac{3}{8}$ by 3, and common sense as well as a rule will tell us that here are 3 times 3 eighths or 9 eighths, which is $1\frac{1}{8}$. This shows us again that to multiply fractions by whole numbers we multiply the numerator or divide the denominator as the case may be.

Dividing a Fraction by a Fraction

But to divide a fraction by a fraction we multiply the two numerators together and also the two denominators together. Suppose we have to make a model $\frac{3}{8}$ inch to the inch, and find a piece that is $\frac{3}{4}$ of an inch in the full-sized piece. We must find how much $\frac{3}{8}$ of $\frac{3}{4}$ is, and we do this by multiplying 3 times 3 and 8 times 4, and getting $\frac{9}{32}$ as the size desired.

It is easier to use signs where we can, so we may as well jot down right here that:

+ means plus or addition,	$4 + 4 = 8$
− means minus or subtraction,	$4 - 2 = 2$
× means times or multiplication,	$4 \times 2 = 8$
÷ means divided by or division,	$4 \div 2 = 2$
= means equals or equal to,	$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$.

Division is also expressed by putting the divisor under the number to be divided, as in a fraction, and is often used in this way: $\frac{\frac{1}{2}}{2} = \frac{1}{4}$, meaning that $\frac{1}{2}$ divided by $2 = \frac{1}{4}$.

In the same way you may want to divide $\frac{7}{16}$ into 4 equal parts, which means that each part will be $\frac{1}{4}$ of $\frac{7}{16}$. Multiplying numerators we have 7, and multiplying denominators we have $4 \times 16 = 64$, or $\frac{7}{64}$ is $\frac{1}{4}$ of $\frac{7}{16}$.

Suppose you have measured up the distance between two surfaces of a jig by using standard distance blocks, and that when you count them up you find one each of the following blocks: $\frac{1}{2}$ inch, $\frac{3}{8}$ inch, $\frac{5}{16}$ inch, and $\frac{7}{64}$ inch, how much is the distance?

It is plain that before we can add these we must get them to the same terms or with the same denominator. As we cannot reduce $\frac{7}{64}$ to the other denominators, we must bring the others to that. To do this, divide each denominator into 64 and multiply the numerator by the result. Starting with the first block, $\frac{1}{2}$ inch, 2 goes into 64, 32 times, and $32 \times 1 = 32$, so $\frac{1}{2}$ inch $= \frac{32}{64}$. In $\frac{3}{8}$, 8 goes into 64, 8 times, and $3 \times 8 = 24$, so $\frac{3}{8}$ inch $= \frac{24}{64}$. In $\frac{5}{16}$, 16 goes into 64, 4 times, and $4 \times 5 = 20$, so $\frac{5}{16} = \frac{20}{64}$. Now they are all in 64ths, and we add numerators, which gives $\frac{32}{64} + \frac{24}{64} + \frac{20}{64} + \frac{7}{64} = \frac{83}{64}$, and dividing 83 by 64 gives $1\frac{19}{64}$ as the total of the blocks. Of course you can add these distances up on a scale to prove them, but it is a good plan to know how to do it on paper or with a piece of chalk on the bench.

It sometimes happens that the denominators will not reduce to any one of them, as would be the case if the fractions were $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{3}$, and $\frac{1}{7}$. In this case it would be necessary to multiply all four denominators together for one that is common to all, as $2 \times 5 \times 3 \times 7 = 210$, which is the least common denominator to them all. Dividing this by each denominator and multiplying the numerators by this,

we have $\frac{1}{2} = \frac{105}{210}$, $\frac{1}{5} = \frac{42}{210}$, $\frac{1}{3} = \frac{70}{210}$, and $\frac{1}{7} = \frac{30}{210}$, so that $\frac{105}{210} + \frac{42}{210} + \frac{70}{210} + \frac{30}{210} = \frac{247}{210} = 1\frac{37}{210}$. This is an exceptional case, but it is well to be ready for emergencies

Dividing by Mixed Numbers

It is sometimes puzzling to divide by mixed numbers as well as by fractions, and yet it often needs to be done. Suppose there is a certain shop which has increased business enough to hire half as many more men as it had before, and now has 180 men. How many had they at first? Here is where a little reasoning comes in, and it is one of the best parts of arithmetic that it makes us think.

After they add one half to their force there is then one and one half as many men as at first, so we know that $180 = 1\frac{1}{2}$ times the original number. So we must divide 180 by $1\frac{1}{2}$.

The easiest way to do this is to turn the $1\frac{1}{2}$ into $\frac{3}{2}$. Then as 180 equals $\frac{3}{2}$, one half must be $\frac{1}{3}$ of 180 or 60, and the original number was $2 \times 60 = 120$. The best of these examples are that we can prove them to be sure we are right. If the shop has 120 men and we add half as many more, we then have $120 + 60 = 180$, as in the example, so that we know we are right.

Suppose a man does 350 pieces of work in $1\frac{3}{4}$ hours, how many does he do in an hour?

Reducing $1\frac{3}{4}$ to quarters we have $\frac{7}{4}$. If $350 = \frac{7}{4}$, then $\frac{1}{4}$ equals $\frac{1}{7}$ of 350 or 50, and $\frac{4}{4}$ or one $= 4 \times 50 = 200$ pieces per hour. This can be proved in the same way as the other.

EXAMPLES

What will a man get for 16½ hours' work at $2.75 per day of 10 hours? *Ans.* $4.53⅞.

How much more will he receive if the work is overtime at "time and a quarter"? *Ans.* 5.67\frac{3}{16}$; 1.13\frac{7}{16}$ more.

If the time cards for a certain piece of work show 2 hours and 10 minutes lathe work, 3 hours and 15 minutes planing, 1 hour and 20 minutes vise work, what will it cost at $3 per day of 9 hours? *Ans.* $2.25.

Which is the cheapest, to have a 12½ cent an hour man take 13¼ hours on a piece of work, or hire a 17½ cent an hour man who can do it in 9⅓ hours? *Ans.* 12½ cent man will cost $1.65⅝; 17½ cent man $1.63⅓.

If four castings, 108½, 97⅓, 86¼, and 21⅞ pounds are bought at 3⅞ cents a pound, what is the total cost? *Ans.* $12.19¾.

A crank-shaft forging weighs 160⅓ pounds, and when finished only weighs 59½ pounds. If the forging cost 17 cents a pound and the metal turned off can be sold for 2½ cents a pound, what is the net cost of the metal in the crank-shaft? *Ans.* First cost, $27.25⅔. Metal turned off, 100⅚ lbs. value of turnings = 2.52\frac{1}{12}$. Net cost, 24.73\frac{7}{12}$.

CHAPTER II

DECIMAL FRACTIONS

IF you use a micrometer caliper or a scale with 10ths and 100th divisions, you will see at once why common fractions do not entirely fill the bill. Time was, and not so very many years ago either, when the very best workmen measured fine work by the 64th "scant" or "full," or would designate it as $\frac{11}{16}$, and a 32d and a 64th, meaning $\frac{11}{16} + \frac{1}{32} + \frac{1}{64}$ or $\frac{47}{64}$, "scant" or "full" as the case might be. But such measurements are clumsy in spite of the fact that they made remarkably close measurements in that way, and every modern machinist must know how to use the micrometer and understand decimal fractions or decimals.

Decimal fractions are written without a denominator, but the denominator is always understood to be 10 or a multiple of 10. The name comes from the Latin word "decem" meaning "ten." Instead of a denominator the number is written with a period in front of it, and the numbers after the period show the value of the decimal. The first number to the right of the decimal point is called tenths, the second place hundredths, the third thousandths, the fourth ten-thousandths, etc., this being as fine as most machine measurements go. Bearing this in mind, we see that $.1 = \frac{1}{10}$, $.01 = \frac{1}{100}$, $.001 = \frac{1}{1000}$, and $.0001 = \frac{1}{10000}$. This shows that every cipher we put to the right of the deci-

mal point, and between the point and the number, divides the number by 10. So to divide .25 by 10 we simply put a cipher ahead of the two and make it .025. Ciphers the other side of the point do not affect it, as 0.1 is the same as .1. Some use a cipher in front of the decimal point to show that it is less than a whole number. Ciphers to the right of the decimal do not affect its value if used, as it is evident that $.10 = \frac{10}{100}$, which is the same as $\frac{1}{10}$.

Whole numbers and decimals are used together the same as mixed numbers in common fractions, such as 2.5, which reads $2\frac{5}{10}$ and the same as $2\frac{1}{2}$.

Suppose the drawing called for a piece $\frac{3}{8}$ inch thick and you wanted to measure this with a micrometer to get it exact. The micrometer does not measure common fractions so it is necessary to change $\frac{3}{8}$ to a decimal. To do this, divide the numerator by the denominator. But how can we divide 3 by 8? By putting a decimal point after the 3 and adding ciphers after it like this:

$$\begin{array}{r} 8\overline{)3.000} \\ .375 \end{array}$$

and put the decimal point in front of as many figures in the answer as you use ciphers in the dividend. This shows that $\frac{3}{8} = .375$, which is $\frac{375}{1000}$, so we set the micrometer at .375 and know it is the same as $\frac{3}{8}$.

Decimals can be added, subtracted, multiplied, and divided the same as whole numbers by simply watching the decimal points. Suppose the opening of a die is measured by three gage blocks, .875, .3125, and .125 inch. To add these, treat them the same as whole numbers, except that the decimal points must be placed under each other, as:

$$\begin{array}{r} .875 \\ .3125 \\ .125 \\ \hline 1.3125 \end{array}$$

and add up the same as whole numbers. The decimal point is placed between the 1 and the 3, because there are only four figures in any of the decimals, and we point off from the right as many places as the largest number of decimals in the figures added.

Subtracting is done in the same way, as .125 from .375 is

$$\begin{array}{r} .375 \\ .125 \\ \hline .250 \end{array}$$

This is a little more difficult when the smaller number has the most figures, as in taking .3125 from .75, as

$$\begin{array}{r} .75 \\ .3125 \\ \hline .4375 \end{array}$$

In this case we add, either actually or in our mind, ciphers to make up the number. Then we say, 5 from 10 = 5, 2 from 9 = 7, 1 from 4 = 3, and 3 from 7 = 4. The point goes before the 4 because there are four places in the longest decimal. There is hardly a place where a little thought and common sense helps more than in pointing off. In the case just given it is very evident that it should not be between the 4 and 3, as the answer cannot be more than either of the numbers. Neither could it be .04375 because it shows that .3 from .7 must be .4 and not .04. A little care in such cases will prevent many mistakes.

Multiplying decimals is very easy as it is exactly like whole numbers, and we point off as many places as there are decimals in both the numbers. It will also be seen that the result is the same as with common fractions. Or $.5 \times .5 = .25$, which is just the same as $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, as we found before.

Multiplying .25 by .25 we have

$$\begin{array}{r} .25 \\ \underline{.25} \\ 125 \\ \underline{50} \\ .0625 \end{array}$$

Here we only have three whole numbers in the answer and, it becomes necessary to put a cipher in front of the 625 to make the four places. Here again a little care helps out, for we know when we multiply two numbers less than 1 the answer is less than either number, and that .625 would very clearly be wrong and .00625 would be altogether too small. Another thing to watch is the multiplying numbers with a lot of ciphers, as

$$\begin{array}{r} 1.2575 \\ \underline{.0021} \\ 12575 \\ \underline{25150} \\ .00264075 \end{array}$$

Here, although one number is over 1, the answer is very much less than 1, because both numbers together have eight decimal places, and the answer had only six before adding the two extra ciphers. While this seems very small we can look at it in another way. This is to say

that .0021 is multiplied by a little more than 1.25 or $1\frac{1}{4}$, and $1\frac{1}{4}$ times .0021 is just about .0026.

Decimals are divided the same as whole numbers, taking care to point off in the answer only as many places as the decimals in the dividend exceed those in the divisor, and is just the reverse of multiplication in every way. Suppose you have a block of steel which figures up to 21.25 cubic inches, and that it weighs 5.9925 pounds, how much per cubic inch does it weigh? Clearly, we must divide the weight by the number of cubic inches, as

```
21.25)5.99250(.282
      4 250
      1 7425
      1 7000
         4250
         4250
```

In order to carry the answer to third decimal place or thousandths, it was necessary to add a cipher to the dividend, and this made five decimal places in the dividend and only two in the divisor, so we point off the difference, or three places in the answer. If we had not added the cipher the answer would have been .28 +, as then there would be but two decimal places in excess of the divisor.

Decimals can also be divided by whole numbers, as in the case of finding the depth of a screw thread. If it is a V-thread we can find out, either by a table or by figuring, that the depth is .866 of an inch, for a pitch of one to the inch, or we can say it is .866 of the pitch. Then for 10 threads to the inch the depth will be .866 divided by 10, which we can do by simply moving the decimal point one

place to the left, as .0866. Or we can perhaps show this more clearly in the case of 16 threads to the inch. Divide .866 by 16, just as though .866 was a whole number, as

```
16).866(.054
    80
     66
     64
      2
```

and as there are three decimal places in the dividend and none in the divisor, there must be three in the answer, so we put a cipher before the first figure, making the answer .054.

It is handy to remember that you can prove any example you do by working it backwards. In addition, subtract one of the numbers from the answer, and it will give the other one if only two are added. If more, add part of them together, subtract from the answer and it will give the sum of all the other numbers.

In subtraction, add the smaller number to the answer and it will give the larger number, or subtract the answer from the larger number and it will give the smaller number.

In multiplication, divide the answer by one of the numbers and it will give the other.

In division, multiply the answer by the divisor and you get the dividend, or divide the dividend by the answer and get the divisor.

Also remember that moving the decimal point to the right multiplies the number by ten for every place moved, and moving it to the left divides it by ten for every place moved.

DECIMAL EQUIVALENTS OF FRACTIONS OF AN INCH. (ADVANCING BY 8THS, 16THS, 32NDS AND 64THS.)

8ths	32nds	64ths	64ths
$\frac{1}{8}$ = .125	$\frac{1}{32}$ = .03125	$\frac{1}{64}$ = .015625	$\frac{33}{64}$ = .515625
$\frac{1}{4}$ = .250	$\frac{3}{32}$ = .09375	$\frac{3}{64}$ = .046875	$\frac{35}{64}$ = .546875
$\frac{3}{8}$ = .375	$\frac{5}{32}$ = .15625	$\frac{5}{64}$ = .078125	$\frac{37}{64}$ = .578125
$\frac{1}{2}$ = .500	$\frac{7}{32}$ = .21875	$\frac{7}{64}$ = .109375	$\frac{39}{64}$ = .609375
$\frac{5}{8}$ = .625	$\frac{9}{32}$ = .28125	$\frac{9}{64}$ = .140625	$\frac{41}{64}$ = .640625
$\frac{3}{4}$ = .750	$\frac{11}{32}$ = .34375	$\frac{11}{64}$ = .171875	$\frac{43}{64}$ = .671875
$\frac{7}{8}$ = .875	$\frac{13}{32}$ = .40625	$\frac{13}{64}$ = .203125	$\frac{45}{64}$ = .703125
	$\frac{15}{32}$ = .46875	$\frac{15}{64}$ = .234375	$\frac{47}{64}$ = .734375
16ths.			
$\frac{1}{16}$ = .0625	$\frac{17}{32}$ = .53125	$\frac{17}{64}$ = .265625	$\frac{49}{64}$ = .765625
$\frac{3}{16}$ = .1875	$\frac{19}{32}$ = .59375	$\frac{19}{64}$ = .296875	$\frac{51}{64}$ = .796875
$\frac{5}{16}$ = .3125	$\frac{21}{32}$ = .65625	$\frac{21}{64}$ = .328125	$\frac{53}{64}$ = .828125
$\frac{7}{16}$ = .4375	$\frac{23}{32}$ = .71875	$\frac{23}{64}$ = .359375	$\frac{55}{64}$ = .859375
$\frac{9}{16}$ = .5625	$\frac{25}{32}$ = .78125	$\frac{25}{64}$ = .390625	$\frac{57}{64}$ = .890625
$\frac{11}{16}$ = .6875	$\frac{27}{32}$ = .84375	$\frac{27}{64}$ = .421875	$\frac{59}{64}$ = .921875
$\frac{13}{16}$ = .8125	$\frac{29}{32}$ = .90625	$\frac{29}{64}$ = .453125	$\frac{61}{64}$ = .953125
$\frac{15}{16}$ = .9375	$\frac{31}{32}$ = .96875	$\frac{31}{64}$ = .484375	$\frac{63}{64}$ = .984375

DECIMAL EQUIVALENTS OF FRACTIONS OF AN INCH. (ADVANCING BY 64THS.)

$\frac{1}{64}$ = .015625	$\frac{17}{64}$ = .265625	$\frac{33}{64}$ = .515625	$\frac{49}{64}$ = .765625
$\frac{1}{32}$ = .03125	$\frac{9}{32}$ = .28125	$\frac{17}{32}$ = .53125	$\frac{25}{32}$ = .78125
$\frac{3}{64}$ = .046875	$\frac{19}{64}$ = .296875	$\frac{35}{64}$ = .546875	$\frac{51}{64}$ = .796875
$\frac{1}{16}$ = .0625	$\frac{5}{16}$ = .3125	$\frac{9}{16}$ = .5625	$\frac{13}{16}$ = .8125
$\frac{5}{64}$ = .078125	$\frac{21}{64}$ = .328125	$\frac{37}{64}$ = .578125	$\frac{53}{64}$ = .828125
$\frac{3}{32}$ = .09375	$\frac{11}{32}$ = .34375	$\frac{19}{32}$ = .59375	$\frac{27}{32}$ = .84375
$\frac{7}{64}$ = .109375	$\frac{23}{64}$ = .359375	$\frac{39}{64}$ = .609375	$\frac{55}{64}$ = .859375
$\frac{1}{8}$ = .125	$\frac{3}{8}$ = .375	$\frac{5}{8}$ = .625	$\frac{7}{8}$ = .875
$\frac{9}{64}$ = .140625	$\frac{25}{64}$ = .390625	$\frac{41}{64}$ = .640625	$\frac{57}{64}$ = .890625
$\frac{5}{32}$ = .15625	$\frac{13}{32}$ = .40625	$\frac{21}{32}$ = .65625	$\frac{29}{32}$ = .90625
$\frac{11}{64}$ = .171875	$\frac{27}{64}$ = .421875	$\frac{43}{64}$ = .671875	$\frac{59}{64}$ = .921875
$\frac{3}{16}$ = .1875	$\frac{7}{16}$ = .4375	$\frac{11}{16}$ = .6875	$\frac{15}{16}$ = .9375
$\frac{13}{64}$ = .203125	$\frac{29}{64}$ = .453125	$\frac{45}{64}$ = .703125	$\frac{61}{64}$ = .953125
$\frac{7}{32}$ = .21875	$\frac{15}{32}$ = .46875	$\frac{23}{32}$ = .71875	$\frac{31}{32}$ = .96875
$\frac{15}{64}$ = .234375	$\frac{31}{64}$ = .484375	$\frac{47}{64}$ = .734375	$\frac{63}{64}$ = .984375
$\frac{1}{4}$ = .25	$\frac{1}{2}$ = .50	$\frac{3}{4}$ = .75	

As it is often necessary to change the common fractions of an inch into decimals and back again, the preceding table is given as being very conveniently arranged, and as a saver of time in figuring it out whenever you want to know what $\frac{3}{8}$ is in decimals, or what fraction is equivalent to .375 of an inch.

EXAMPLES

A foreman gave the toolmaker three distance blocks, to make a snap gage that would just fit when the three blocks were put together. The blocks were 0.625, 0.28125, and 0.015625 inches, what was the size of the snap gage? *Ans.* 0.921875 inches.

What are the decimal equivalents to $\frac{51}{128}$ and $\frac{19}{64}$? *Ans.* 0.39843 and 0.296875.

If the correct size of a reamer is 0.796875 and the toolmaker finds it is only 0.78929, how much too small is it? *Ans.* 0.007585 too small.

When the limit of accuracy is 0.005 inch, what is the largest and the smallest diameter allowable, if the nominal size is 1.175 inch. *Ans.* 1.1725 and 1.1775.

If a reamer is 1.1875 on the small end and 1.375 on the large end, and the tapered portion is 6 inches long, what is the taper per foot? *Ans.* 0.375 inch per foot.

CHAPTER III

CANCELLATION

JUST as we use some tools to make the work easier and quicker, so we can use methods or short cuts which will save time. One of these is called cancellation, and it will save time in both multiplication and division. It comes especially handy with formulas, such as the one for horsepower where we have

$$H.P. = \frac{P \times L \times A \times N}{33000}.$$

where pressure, P, = 100 pounds, length of stroke, L, = 3 feet, area of piston, A, = 180 square inches, and N = 200 strokes per minute. So we have,

$$H.P. = \frac{100 \times 3 \times 180 \times 200}{33000}.$$

Instead of multiplying all the numerators together, we see how much we can reduce both numerator and denominator by dividing both of them by any number which will go evenly into both.

We can begin by dividing 33000 by 100, and crossing out the 100 above the line, which leaves 330 as the denominator. Divide it again by 3 and get 110. Divide 180 and 110 by 10 and get 18 and 11.

$$H.P. = \frac{\cancel{1}\cancel{0}\cancel{0} \times \cancel{3} \times 18\cancel{0} \times 200}{\underset{11\cancel{0}}{\cancel{3}\cancel{3}\cancel{0}\cancel{0}\cancel{0}}}.$$

Nothing will divide 11 and this will not go in to 18 or 200, so we have reached the limit. Now all we have to do is to multiply $18 \times 200 = 3600$ divided by 11, which gives 327 horse-power.

Let us suppose a case of a train of driving gears where the driving gears are 24, 36, 48, 55, and the driven 30, 40, 44, and 60 teeth. The driving gears go above the line and the driven gears below, so we have

$$\text{Ratio of speeds} = \frac{24 \times 36 \times 48 \times 55}{30 \times 40 \times 44 \times 60}.$$

Starting with the first pair, divide by the largest number that will go in both, which is 6, leaving 4 and 5. The 4 will go in 40 and leave 10. In 36 and 60, 12 will go 3 and 5 times. In 55 and 44 we can divide by 11, leaving 5 and 4. The 4 will go in 48, 12 times, the 5's cancel each other, 12 and 10 are divisible by 2, leaving 6 and 5.

$$\frac{\overset{\cancel{4}}{\cancel{24}} \times \overset{3}{\cancel{36}} \times \overset{\overset{6}{\cancel{12}}}{\cancel{48}} \times \overset{\cancel{5}}{\cancel{55}}}{\underset{5}{\cancel{30}} \quad \underset{\underset{5}{\cancel{10}}}{\cancel{40}} \quad \underset{\cancel{4}}{\cancel{44}} \quad \underset{\cancel{5}}{\cancel{60}}}.$$

This leaves us only $\frac{3 \times 6}{5 \times 5} = \frac{18}{25}$, showing that the whole train is equivalent to an 18-tooth driving gear and a 25-tooth driven gear.

This will be found extremely useful in all sorts of calculations, and a little practice will make it easy to do many examples without any division except what can be done in your head.

Pulleys make one of the cases where cancellation comes in very handy as a time saver and a few examples will show how it saves figuring. Take as an example a line-shaft running 200 revolutions, pulley on the line-shaft 30 inches, pulley on a grinder 6 inches, how fast will the grinder run? An easy rule fits this case, and is: multiply the diameter of the driving pulley by its speed and divide by the diameter of the driven pulley, which would be $30 \times 200 = 6000$ and divided by $6 = 1000$ revolutions for the grinder.

We learned in common fractions that the placing of figures in this way meant that the numerator was divided by the denominator, so in cancellation we place the numbers that are to be multiplied together above the line, and the number or numbers we divide by below the line, like this:

$$\frac{30 \times 200}{6}.$$

Then, instead of multiplying 30 by 200 we see if any of the numbers below the line will divide into any of those above the line evenly, or the other way, just as it happens. In this case 6 will divide evenly into 30, so we cross out the 6 and put a 5 over the 30, to show that the 30 has been reduced to 5 by crossing out the 6 below the line, like this:

$$\frac{\overset{5}{\cancel{30}} \times 200}{\cancel{6}} = 1000.$$

Then we only have to multiply 5 times 200, which gives 1000 without using a pencil at all. This is such a simple case that the saving is very little indeed, but it often happens that it reduces the work wonderfully. Suppose we

have an example where 13, 20, 16, and 21 have to be multiplied together, and divided by the product of 39, 5, 7, and 8. This would mean a lot of multiplying and dividing, but by the cancellation plan the work is easy, and we write it like this:

$$\frac{13 \times 20 \times 16 \times 21}{39 \times 5 \times 7 \times 8}.$$

Looking for numbers that will cancel we find that 13 goes into 39 just 3 times, so cross out both 13 and 39 and put a 3 in place of 39. Next 5 goes in 20 just 4 times, 7 goes in 21 just 3 times, and 8 goes into 16 just 2 times, so we have:

$$\frac{\cancel{13} \times \overset{4}{\cancel{20}} \times \overset{2}{\cancel{16}} \times \overset{3}{\cancel{21}}}{\underset{3}{\cancel{39}} \times \cancel{5} \times \cancel{7} \times \cancel{8}}.$$

Looking it over we find a 3 above the line and another below, so we cross out both of them, which leaves only 4 and 2 above the line, and nothing at all below, so we now have:

$$\frac{\cancel{13} \times \overset{4}{\cancel{20}} \times \overset{2}{\cancel{16}} \times \overset{\cancel{3}}{\cancel{21}}}{\underset{\cancel{3}}{\cancel{39}} \times \cancel{5} \times \cancel{7} \times \cancel{8}} = 4 \times 2 = 8$$

which is certainly much easier and quicker than multiplying and dividing in the regular way.

Here is a lathe counter-shaft with two 10-inch pulleys. The forward pulley is to run 300, the backing pulley $1\frac{1}{2}$ times as fast, or 450 turns for a quick return. The line-shaft runs 150 revolutions, what pulleys must go on the line-shaft? We have the diameters and speeds of the

driven pulleys, and the speed of the drivers to find their diameters; so, as we must divide by the speed of the drivers, we put this below the line and the driven above the line. Diameters are both 10 inches, so we say $\frac{10 \times \overset{2}{\cancel{300}}}{\cancel{150}} = 20$ inches for forward pulley on line-shaft; and $\frac{10 \times \overset{3}{\cancel{450}}}{\cancel{150}} = 30$ inches for backing pulley on line-shaft.

Suppose, however, that a shaft *A* runs 200 turns a minute, and the pulley *B* on *A* is 12 inches in diameter. This drives an 8-inch pulley *D* on a shaft *C*; and pulley *E*, also on shaft *C*, is 18 inches in diameter, while pulley *F* on a shaft *G* is only 6 inches in diameter, how fast will shaft *G* run?

Separate the driving pulleys from the driven, no matter how many there are of each, and as the answer is to be the speed of the driven pulleys, the diameters of the driven pulleys make the denominator, while the diameters and speed of driving pulleys go above the line. This makes it

$$\frac{200 \times 12 \times 18}{8 \times 6} = \text{speed of shaft } G,$$

and as 6 will go in 18 just 3 times we cancel these. Then 4 will go in 8 twice and in 12 three times, so we cancel these. The 2 will go in 200 evenly, making it 100, and we now have $3 \times 3 \times 100 = 900$, as follows:

$$\frac{\overset{100}{\cancel{200}} \times \overset{3}{\cancel{12}} \times \overset{3}{\cancel{18}}}{\underset{\cancel{2}}{\cancel{8}} \times \cancel{6}} = 900.$$

This solves the problem without bothering to find the speed of the intermediate shaft C, which was not asked, but which can be found as before, and will be

$$\frac{\overset{25}{\cancel{200}} \times 12}{\cancel{8}} = 300 \text{ revolutions.}$$

Suppose the engine should run away and get up to 200 revolutions per minute, what would be the speed of the rim of the fly-wheel (and of the belt) if the wheel is 10 feet in diameter, also what is the surface speed of the bearings if the shaft is 12 inches in diameter at the journal?

Multiply the diameter by the constant 3.1416 to get the circumference, which is $10 \times 3.1416 = 31.416$ feet, and at 200 revolutions we get 200×31.416 or 6283.2 feet, over a mile a minute, which is too fast for a cast-iron wheel. The bearing is 1×3.1416 or 3.1416 feet for each revolution, and 628.32 feet surface speed for the bearings, just one tenth the fly-wheel speed.

EXAMPLES

Cancel the following down to the lowest terms:

1. $\dfrac{2 \times 6 \times 18 \times 3}{3 \times 7 \times 12 \times 8}$.

2. $\dfrac{7 \times 4 \times 9 \times 3}{8 \times 12 \times 7 \times 8}$.

3. $\frac{1}{2} \times \frac{3}{4} \times \frac{17}{3} \times \frac{11}{34}$.

4. $\dfrac{84 \times 110 \times 36 \times 10}{33000 \times 6}$.

Ans. $1 = \frac{9}{14}$, $2 = \frac{9}{64}$, $3 = \frac{11}{16}$, $4 = \frac{84}{5}$.

CHAPTER IV

RATIO OR PROPORTION

RATIO and proportion is the same as the old "rule of three" of our forefathers and is one of the handiest rules in arithmetic, if it is understood.

One of its uses is in selecting change gears for screw cutting. Calling the lead screw 4 pitch and the thread to be cut 8 pitch, we say the "ratio" is 4 to 8, or 1 to 2, because 4 is one half of 8. We know that if we could use gears with 4 and 8 teeth the right thread would be cut, but we have no gears with such a small number of teeth; so we must find others having the same ratio or proportion to each other. This is why we multiply both by the same number in all cases to find what gears we can use.

Multiplying 4 and 8 by 6 gives 24 and 48, or by 7 gives 28 and 56. If the gears run in fives instead of fours, that is, vary five teeth, as 25, 30, 35, etc., multiply 4 and 8 by 5, giving 20 and 40. But 20 is smaller than we usually have, and we must find two other gears, with the teeth divisible by 5, which are in the same ratio, or of which one has twice as many teeth as the other. Multiplying by 10 gives 40 and 80; by $7\frac{1}{2}$ gives 30 and 60; by $6\frac{1}{4}$ gives 25 and 50, all being in the same proportion or ratio. You can multiply by any number that will give gears that you have, and as long as you multiply both by the same number the ratio or proportion is unchanged.

Another example is in the method of laying out a square corner, by what is called 6, 8, and 10 rule. This means that any triangle having sides of 6, 8, and 10 inches, or in that ratio, is always a right-angled triangle. The sides can be 6, 8, and 10 feet or miles, and the angle will be the same. Or it can be 3, 4, and 5; $1\frac{1}{2}$, 2, and $2\frac{1}{2}$; 9, 12, and 15; 30, 40, and 50 inches, feet, or miles; the ratio is the same in all these cases. As long as we multiply or divide all the factors or parts by the same number, we have not disturbed the proportion.

If a casting weighs 7 pounds and we want to know how much 13 castings would weigh, we simply multiply 13 by 7 and get 91 pounds. But if you are told that 13 castings weigh 91 pounds and want to find how much 7 will weigh, it looks a little harder. But there is an easy way to work out any of these problems with very few figures. First draw a horizontal line and then ask yourself whether the answer wants to be greater or less than the figure you already have. In this case the 7 castings will weigh less than the 13, so put the line and figures like this:

$$\frac{7 \times 91}{13} = \frac{637}{13} = 49,$$

or it would have been easier to do this by dividing 13 into 91 and canceling, like this:

$$\frac{7 \times \overset{7}{\cancel{91}}}{\cancel{13}} = 49,$$

leaving only $7 \times 7 = 49$.

This works out very nicely in figuring out belt speeds, pulley diameters or gears, and will save lots of time if used

carefully. Suppose we have a nest of compound gears, with the driving gears having 100, 50, and 30 teeth, and the driven gears 60, 40, and 20 teeth; how fast will the driven shaft turn if the first driver turns 100 times a minute? The easiest way is to put all the driving gears on top of the line and the driven gears below, as follows:

$$\frac{100 \times 50 \times 30}{60 \times 40 \times 20},$$

Canceling, we divide 60 by 30, crossing out both numbers and put down 2; 20 into 100 gives 5, and dividing 40 and 50 by 10 (canceling ciphers) gives 4 and 5. This leaves 5×5 over the line and 2×4 below, as follows:

$$\frac{\overset{5}{\cancel{100}} \times 5\cancel{0} \times \cancel{30}}{\underset{2}{\cancel{60}} \times 4\cancel{0} \times \cancel{20}} = \frac{5 \times 5}{2 \times 4} = \frac{25}{8}.$$

This shows that the driven shaft will turn $\frac{25}{8}$ times as fast as the driver. If the gears had been reversed and the smaller ones the drivers, the numbers would also be reversed and the larger ones placed below the line. Then the driven shaft would turn $\frac{8}{25}$ as fast as the driver. In the first case the driven shaft will run

$$\frac{25 \times 100}{8} = \frac{2500}{8} = 312\tfrac{1}{2} \text{ revolutions.}$$

In the second it would run

$$\frac{8 \times 100}{25} = \frac{800}{25} = 32 \text{ revolutions per minute.}$$

Suppose you earn \$7 making 35 pieces of work, how many

pieces can you make for $11? As you know it will be more, just put the example like this:

$$\frac{11 \times 35}{7}$$

and cancel as follows:

$$\frac{11 \times \overset{5}{\cancel{35}}}{\cancel{7}} = 55 \text{ pieces.}$$

The school books always give proportion like this:

2 : 4 : : 8 : 16

which is read, "as 2 is to 4 so is 8 to 16." The four numbers are called the four "terms," first, second, third, and fourth, and the first multiplied by the fourth equals the second multiplied by the third. So to find the first or fourth terms we multiply the second and third together and divide by the one we have, or reverse this if the first and fourth are given and one of the others missing.

EXAMPLES

If a worm runs 260 turns a minute and the worm wheel 13 turns a minute, what is the ratio of reduction? *Ans.* 20 to 1.

Three gears mesh together and drive different shafts. The driver has 100 teeth and runs 80 turns a minute, the next gear has 80 teeth and the last 60 teeth. What is the ratio between the speed of the shafts? *Ans.* First runs 80, second 100, third $133\frac{1}{3}$ revolutions per minute.

If a mixture has copper 3 parts, lead 2 parts, and tin 1 part, what amount of each will it take to make a 60-pound casting? *Ans.* Copper, 30 pounds; lead, 20 pounds; tin, 10 pounds.

CHAPTER V

PERCENTAGE

PERCENTAGE is very closely related to decimals and is very easy as well as very useful. It may be called a decimal system with one hundred as the base, and comes from two words, per, centum: per — by, and centum — hundred; by the hundred.

So one per cent means one one-hundredth of whatever we are talking about. If a raise in pay of 10 per cent is made, it means that one tenth of the present amount will be added to it. If the rate is \$3 per day, 10 per cent of this is $\frac{1}{10}$ of \$3 or 30 cents, so that the new rate is \$3.30. If a formula for babbitt metal is 82 per cent lead, 15 per cent tin, and 3 per cent copper, it does not mean that there must be 82 pounds of one, etc., but that $\frac{82}{100}$ of whatever the amount is, is to be lead.

If we have a motor with 90 per cent efficiency, it means that we are getting 90 parts of work out of a possible hundred, or in the case of a 100 horse-power motor we get out 90 horse-power by putting 100 horse-power of work into the motor at the wires. In other words, 10 horse-power is lost in friction and the electrical losses.

We often hear belt slippage given as 2 per cent, which means that for every 100 feet the belt should travel, it only travels 98 feet, the slippage being 2 feet, or, if the pulleys

are the same size on both driver and driven, and the driver runs 100 revolutions per minute, the driven shaft will only run 98 revolutions, owing to the 2 per cent slippage.

The only difficulty in figuring percentage is to know what number to use as the base or principal. If you have \$200 in the bank at 4 per cent interest, multiply 200 by $.04 =$ \$8.00, as the interest for one year. If you leave this in the bank the interest is no longer figured on \$200, but on \$208, so that the next year the interest is \$208 $\times$ .04 = \$8.32.

Going, back to the question of the 10 per cent raise in pay, from \$3 to \$3.30, we see the result of the difference in the base for a 10 per cent cut would leave us worse off than before; for $3.30 \times .10 = .33$ cents, which would leave only \$2.97 instead of \$3, because we are dealing with a larger sum as the base. If we offer a man \$400 for a piano, for which he asks \$500, we must raise our offer 25 per cent to meet him, as $400 \times .25 = 100$, while he can meet us by cutting his price only 20 per cent, as $500 \times .20 = 100$, showing the difference in the way we figure.

If a metal is 60 per cent lead, 10 per cent copper, and 30 per cent tin, and the lead weighs 12 pounds, how much tin and copper is there, and how much does it all weigh?

As the lead weighs 12 pounds, and this is 60 per cent, one per cent = $\frac{1}{60}$ of 12, or $\frac{12}{60}$ of a pound. The copper weighs 10 times this, or $\frac{120}{60}$, or 2 pounds, and the tin 30 times or $\frac{360}{60} = 6$ pounds. Or we could say that, as the lead weighed 12 pounds, and there was $\frac{1}{6}$ as much copper as lead, this must be 2 pounds, and as the tin is $\frac{1}{2}$ the lead, this will be 6 pounds, making $12 + 2 + 6 = 20$ pounds in all.

Or if by adding 10 per cent to the men in the shop you would then have 220, how many have you now?

This puts a different light on it, but while you do not know the number now employed, you know that it equals 100 per cent, whether it is one man or 200. So if you add 10 per cent and you then have 110 per cent of the original force. So 220 = 110 per cent and one per cent equals 220 divided by 110, or 2, and $100 \times 2 = 200$, the original number.

This shows that percentages may be very misleading. A shop employing only 4 men would increase its force 25 per cent if it hired the fifth man, while a shop of 100 men would have to employ 25 more men to equal this percentage, and a shop with 2000 men would have to hire 500 new men to increase 25 per cent, yet the percentage is the same in either case.

EXAMPLES

A machinist has been raised from $2.50 to $2.75 per day, what is the percentage of the increase? *Ans.* 10 per cent.

If he is cut 10 per cent from this, what will his daily wage be? *Ans.* $2.47½.

Why is 10 per cent reduction more than the 10 per cent increase?

If we take a contract for $1800, and it costs us $1600, what percentage have we earned on the cost? *Ans.* 12½ per cent.

A job nets us $300 on the contract price, which is a 5 per cent profit on the cost. What was the cost and the contract price? *Ans.* Cost, $6000; contract, $6300.

CHAPTER VI

SPEED OF PULLEYS

ONE of the most common problems which comes up in the shop is about the speed of belts, pulleys, and gears. A new machine is bought which has a counter-shaft that should run 400 revolutions a minute, according to the tag on the counter. What size pulley goes on the line-shaft to give this speed? This depends on the speed of the line-shaft and the size of the pulley on the counter-shaft. If the line-shaft runs at 200 revolutions and the pulley on the counter is 12 inches in diameter, we have enough information for the job.

As the counter must run faster than the line-shaft, it is very plain that the pulley on the line-shaft must be larger than the one on the counter, and as 400 is just double 200, it must be just twice as large, or 24 inches in diameter. This is a case where we can figure it out in our head, but let us see why.

A belt drives by running around on the rim of a pulley, and if the distance around both pulleys is the same they must run at the same speed because the rim of each must run at the same speed. In Fig. 1 it will be seen that when the large pulley turns once around, the rim of the small pulley must travel twice as far, and this holds good whether they turn part of a revolution or 100 revolutions.

Proving this by measuring the distance around the pulley either by using a tape or by rolling a pulley on a bench until it has made one turn, we find that a ... pulley ... inches around, a 14-inch pulley ... inches and a ...-inch pulley ... inches around. This is ... times the

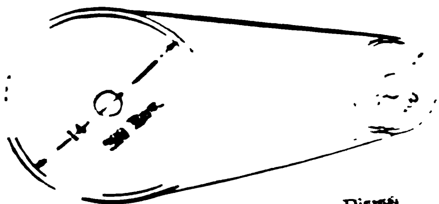
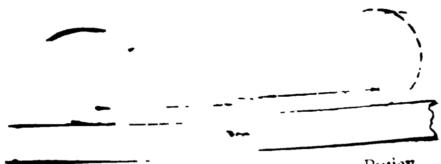


Fig. ... — ... of Different Diam...



... ference of Pulley.

... decimals, 3.1416 is the usually ... called a "constant" because it is ... relation between the diameter ... distance around it, or its circumference. ... periphery or the perimeter. The ... the outside boundary of any shaped ...

... the counter-shaft pulley being 12 inches ... easy to multiply this by 3.1416, and find ... circumference is 3.1416 feet. This must run 400 ... 4 feet per minute. It is plain that the

pulley which drives the counter-shaft must have a circumference that will run at the same speed, and as the line-shaft only runs 200, we must divide 1256.64 by 200 to find the distance around the pulley or the distance moved in one turn of the shaft. This gives 6.2832 feet around the pulley, and dividing by the constant 3.1416 we get 2 feet for the diameter.

When we stop to think that the circumference is always 3.1416 times the diameter, we see that we can just as well use the diameter in each case, and that it is very much easier. This would let us multiply the diameter of counter-shaft pulley by 400, and divide this by 200, getting 2 feet as the right diameter for line-shaft pulley without any decimals whatever.

A little practice will enable any one to figure almost any necessary example in pulley speeds in their head without resorting to the use of paper and pencil at all, and it is very much better to reason out the whole thing than to depend on any rule without knowing why it is used.

A very common way of working these out mentally is to find how many times larger or smaller the driven pulley is and divide or multiply the driving pulley diameter by this number, as the case may be. With the driving pulley 30 inches and the driven pulley only 4 inches it will run 7½ times the speed of the driving pulley.

Having seen the reason why, we can make this into a little rule which may be handy, based on the fact that the diameter of the *driving* pulley multiplied by its speed always equals the diameter of the *driven* pulley multiplied by its speed. So we can say

Having	To Find	Rule
Diameter of driving pulley Diameter of driven pulley Speed of driving pulley	Speed of driven pulley	Multiply diameter of driving pulley by its speed and divide by diameter of driven pulley
Diameter of driving pulley Speed of driving pulley Speed of driven pulley	Diameter of driven pulley	Multiply diameter of driving pulley by its speed and divide by speed of driven pulley
Diameter of driving pulley Diameter of driven pulley Speed of driven pulley	Speed of driving pulley	Multiply diameter of driven pulley by its speed and divide by diameter of driving pulley
Diameter of driven pulley Speed of driven pulley Speed of driving pulley	Diameter of the driving pulley	Multiply diameter of driven pulley by its speed and divide by speed of driving pulley

These cover all the cases of direct driving with only two pulleys, and where there are more, as in belting from line-shaft to counter, and from counter to the machine, it can be worked as two separate problems or combined into one as shown on page 20.

EXAMPLES

With the line-shaft running at 180 turns a minute and the counter-shaft pulley 14 inches in diameter, what size pulley must go on the line-shaft to drive counter 420 a minute? *Ans.* $32\frac{2}{3}$; in practice, 33 inch.

The engine makes 90 turns a minute, and is to drive a dynamo at 1200 through a jack-shaft, which is to run 300. Driving wheel is 10 feet and pulley on dynamo 12 inches. What must the driven and driving pulleys on jack-shaft be?

Ans. Driven pulley on jack-shaft, 36 inches; pulley on jack-shaft driving dynamo, 48 inches.

With a grindstone 7 feet in diameter, how many turns a minute must it run to get a surface speed of 3300 feet per minute, and what pulley must be used on the stone to run from a 15-inch pulley on line-shaft running 200 a minute? *Ans.* Grindstone must run 150 turns and have a 20-inch pulley.

CHAPTER VII

SPEED OF GEARING

THE speed of gearing is figured in just the same way as the speed of belts and pulleys, except that the number of teeth are used instead of diameters. This gives the same results as using the pitch diameter, but as this is less than the outside diameter, the number of teeth is easier to find and is used on that account. If a gear having 20 teeth drives another gear having 40 teeth, it is very plain that when the driving gear makes one turn the driven gear of 40 teeth will only have made half a turn or be running half as fast. The same rules apply as for pulleys, if we substitute teeth for diameter. The number of teeth in the driving gear multiplied by its speed equals the number of teeth in driven gear multiplied by its speed; so to find either the teeth or speed of the driven gear we use the factor we have of the driven gear as the denominator or divisor. Thus if the 20-tooth gear ran 100 turns a minute, the problem would become $\dfrac{\cancel{20} \times \overset{50}{\cancel{100}}}{\underset{2}{\cancel{40}}} = 50$ turns for the 40-tooth gear.

This is especially handy where there is a long train of gears, in which the intermediate shafting is driven by one

gear and drives the next with a different gear. Where it is a direct train the intermediate gears are all idlers, which, if the driver has 20 teeth, simply transmits the motion of 20 teeth to the last gear of the train. If this has 40 teeth as before, it will turn one half as fast as the driver, no matter how many intermediate gears are in the train.

But where a 60-tooth gear drives a 40, and a 60-tooth gear on the same shaft drives an 80, while a 40 drives a 30, and a 40 drives a 24, it is easy to find the speed of each or the speed of the last, whichever you need. If you want the speed of the driven gear, then the teeth of the driven must be the divisor, and calling the speed of the first shaft 80 turns a minute, the second will be $\frac{60 \times \overset{2}{\cancel{80}}}{\cancel{40}} = 120$ turns. The driver here is 60 and the next driven gear has 80 teeth, so we have $\frac{\overset{30}{\cancel{60}} \times \overset{3}{\cancel{120}}}{\underset{\cancel{2}}{\cancel{80}}} = 90$ turns for the third shaft.

Here a 40-tooth gear drives a 30, so the fourth shaft must run $\frac{40 \times \overset{3}{\cancel{90}}}{\cancel{30}} = 120$ revolutions per minute. On this is a 40 driving a 24 on the last shaft, so we have $\frac{\overset{10}{\cancel{40}} \times \overset{20}{\cancel{120}}}{\underset{\cancel{6}}{\cancel{24}}} =$ 200 revolutions per minute.

Now, to get the final speed at once without all these intermediate shafts, we can make this all into one example by

putting the speed of the first driver and the teeth of all the drivers above the line, and the teeth of all the driven gears below the line. This gives the same as before.

$$\frac{\cancel{80} \times \overset{2}{\cancel{60}} \times 60 \times \cancel{40} \times \overset{10}{\cancel{40}}}{\cancel{40} \times \cancel{80} \times \cancel{30} \times \underset{\underset{3}{\cancel{12}}}{\cancel{24}}} = \frac{600}{3} = 200.$$

Suppose this had been reversed, and we had the same gears, but that the last shaft must run 250 turns, how fast must the first shaft run? We now have the speed of the driven, so the driving gears become the divisors and go below the line like this:

$$\frac{\overset{50}{\cancel{250}} \times \overset{2}{\cancel{40}} \times \cancel{80} \times \cancel{30} \times \overset{\cancel{12}}{\cancel{24}}}{\underset{\cancel{5}}{\cancel{60}} \times \underset{\cancel{2}}{\cancel{60}} \times \cancel{40} \times \cancel{40}} = 100$$ turns a minute for the first shaft.

EXAMPLES

The first shaft runs 100 turns a minute and carries an 80-tooth gear, which drives a 60-tooth gear on the second shaft. A 40-tooth gear on this drives a 60-tooth gear on the third. What is the speed of each shaft? *Ans.* 100, $133\frac{1}{3}$, and $88\frac{8}{9}$ revolutions per minute.

What gear will be necessary to drive a second shaft 48 turns a minute if the driver has 72 teeth and runs 114 turns a minute? *Ans.* 171 teeth.

If a gear of 100 teeth running 99 turns a minute drives a gear of 99 teeth, how fast will it run? *Ans.* 100 turns a minute.

CHAPTER VIII

GEARING A LATHE TO CUT ANY THREAD

THE selection of the proper gears for screw cutting is a problem as old as the engine lathe itself, and yet it probably comes up more frequently in the shop than any other, not even excepting the question of turning tapers. Instead of attempting to give any hard-and fast rules at first, it seems better to try to understand just "how" the carriage and tool are moved in screw cutting, so that it will be perfectly clear "why" we make each move in the game.

Simple Gearing

Fig. 1 shows the head of an engine lathe with simple gearing. Gear *E* is solid with the lathe spindle and turns with it in the direction of the arrow. Neither *F* nor *G* is in mesh with *E*, so no motion is given to any of the gears or the lead screw.

Raising gear bracket *I* so as to mesh *G* into *E*, and tracing the turning direction of the various gears, we find that *L* turns in the same direction as the spindle, so that a right-hand thread on the lead screw will move the carriage toward the head and cut a right-hand thread, as is usual. It is easy to trace out the direction in which gears will turn in several ways. One is to remember that every other gear turns in the same direction. This means that *E*, *H*

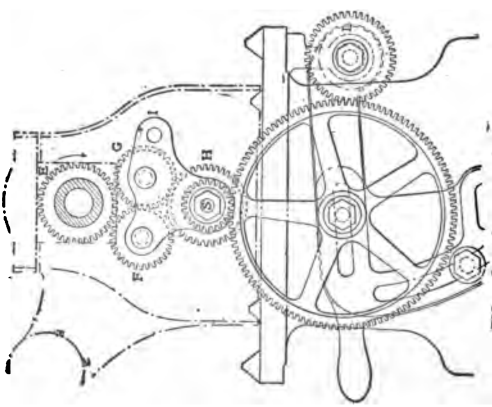


FIG. 1 Screw Cutting with Simple Gears

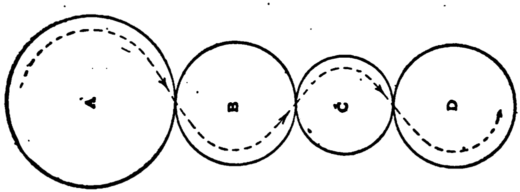


FIG. 2 Following the Motion of Gears in a Train

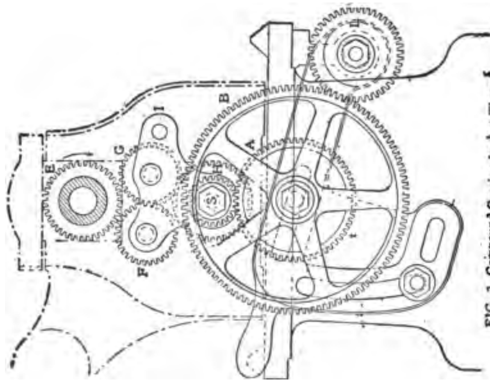


FIG. 3 Compound Gearing for fast Threads

Simple and Compound Lathe Gearing.

and *L* will all turn the same way. The gear *F* is not in the train, but is running idle in this position. Another way is to follow the gears themselves with your eye or with a stick (but don't risk a finger if they are in motion), as shown by the arrows in Fig. 2, following the motion till you come to the end or to the particular gear wanted. The fact that the large intermediate gear does not mesh with *H* does not affect its motion, as the smaller gear is on the same shaft, or what is commonly called the stud.

Reversing the Lead Screw

To cut a left-hand thread, the handle *I* is lowered until gear *F* meshes with *E*. This makes *L* run in the reverse direction, as *E*, *G* and the intermediate run the same way. This is the object of the gear *F*, which is idle when cutting right-hand threads.

Having settled the direction in which the gears run, the next question is, the gears required to produce the right thread on the work. It is easy to see that if the lathe spindle and the lead screw revolve at the same rate, the carriage will advance one thread at each revolution and cut the same thread as the lead screw.

With the gearing shown in Fig. 1, this will not be the case, because the gear on the stud *S* has fewer teeth than the main driving gear *E*, so that the lead screw will turn at a slower rate, and cut a finer thread.

Finding the "True" Thread of Lead Screw

The first thing to do is to see if the stud *S* turns at the same speed as the spindle. This can be done by counting the teeth in *E* and in *H*. These are usually the same, and

if so, the stud turns at the same rate as the spindle and the lathe is geared "even." If these gears are inside the head, and hard to get at, put gears having the same number of teeth on both the stud and the lead screw, and take a cut on an old piece of stock to see what it cuts. If the thread is the same as the lead screw, the gears are "even," as before; if not, call the thread you cut in this way *the true thread of your lead screw* in all cases, and ignore the actual pitch of your lead screw in all calculations. This will save much time and trouble in future.

After you have found the true pitch of your lead screw it is easy to find the gears for any thread as long as the train of gearing remains the same, that is, not compounded in any way.

Calculating the Gears

All you have to do is to multiply both the pitch to be cut and the true pitch of the lead screw by the same number and you get the gears to use.

Call your lead-screw pitch 6, and you want to cut a 10 thread. Multiply both 6 and 10 by 4 and get 24 and 40, or by 5 and get 30 and 50, or by 6 and get 36 and 60. It doesn't matter which pair you use if you put them on the right place.

Just remember that the gear you get by multiplying the pitch of lead screw goes on the stud, and you'll have no trouble. The other, of course, goes on the lead screw. Let us see why, so there will be no need of remembering or taking any one's say so.

The screw to be cut in this case is 10 pitch, slower than the lead screw. So the carriage must move more slowly than to cut a screw the same pitch as the lead screw. This

means the lead screw must revolve more slowly than the spindle; to do this, the larger gear must go on the lead screw.

Suppose you select gears with 30 and 50 teeth and put the 30 on the stud. Then every revolution of the stud will turn all the gearing in the train just 30 teeth, which will revolve the lead screw $\frac{30}{50}$, or $\frac{3}{5}$ of a revolution, which is correct for a 10 thread with a 6-pitch lead screw. But it isn't necesssary to bother with this figuring unless you want to prove the "why" of it to yourself.

A Few Simple Rules

If you want this in a little rule or rules, you can say; multiply true pitch of lead screw and pitch of thread to be cut by any number that will give two gears that you have for that lathe.

Put the gear obtained by multiplying the thread to be cut, on the lead screw.

Or, if the pitch of the thread to be cut is *faster* than the lead screw, the smallest gear goes on the screw. If slower than lead screw, smallest gear goes on the stud. But — don't apply these rules unless you know what thread the lathe will cut, with even gears on both stud and lead screw.

Compounding the Gears

So far it has been plain sailing with a direct train of gears, but when you begin to double up or compound it is necessary to keep your weather eye open for squalls. They don't arrive if you take time to be sure you are right, but lie in wait for the fellow who "knows it all" or who "never makes a mistake."

Compound gearing is necessary to give the lathe a large range of threads, as it isn't practical to use a 160-tooth gear to cut a 40 thread, as would be necessary with a 6-pitch lead screw and a 24 gear on the stud. So "compound" or change the gearing between stud and the lead screw, as in Fig. 3.

Where the Difference Comes In

Here all the gearing between the spindle and the stud is the same as Fig. 1. But instead of the stud gear driving the same large gear as drives the gear on lead screw, as in Fig. 1, the stud gear drives gear *A*, and fastened to this, and turning with it, is gear *B*, which drives the lead-screw gear *L*.

The gear *A* is one half the diameter of *B*, and has one half as many teeth. As they both turn together, one revolution will move *A* 60 teeth and *B* twice this, or 120 teeth; and as *B* drives *L*, then *L* will be driven 120 teeth also, or twice as fast as though it were driven direct from *S*, by both meshing into the same intermediate gear, as in Fig. 1. The compounding in this case is "geared up" to cut a rapid thread, such as $1\frac{1}{2}$ to the inch. With straight gears this would call for $16\times1\frac{1}{2}=24$; and $16\times6=96$. If we have no 96 gear we gear up the compound attachment, as shown, and use a 48 gear on the stud, the 24 on the lead screw.

To "gear down" for cutting a finer thread, as a 40, the stud *S* would drive the large gear *B*, while *A* would drive lead-screw gear *L*, through an intermediate. This brings us to the form of compound gearing shown in Fig. 4, which is quite common on modern lathes, and is also somewhat puzzling unless you pick the arrangement to pieces. So this is the next step.

A Compound Cone of Gears

The upper sets of gears A, B, C, are loose on the shaft or pin, except when made to drive with it by the feather or key K, shown engaged in A. Gears B and C are now running idle. Gears D, E and F are keyed solidly together. As shown, the lathe is simple geared from A, through D and G, to lead screw H. Calling A 24 teeth, one turn will revolve D the same number of teeth, which also moves G 24 teeth, and revolves lead-screw gear H once, as this has 24 teeth. So far this is simple gearing, as shown in outline by Fig. 5.

The next change is to move gears G and H in toward the head, so as to mesh in with E. This is shown in Fig. 6. Now A, with 24 teeth, drives D, having 48 teeth. E, with 36 teeth, drives the intermediate gear G, and screw gear H.

When A revolves one turn, or moves 24 teeth, D must also move 24 teeth, or one half a turn. But E, also making half a turn, moves 18 teeth, and this is transmitted to gear H, showing that "gearing down" takes place as we move gears G and H to the smaller diameters of the cone.

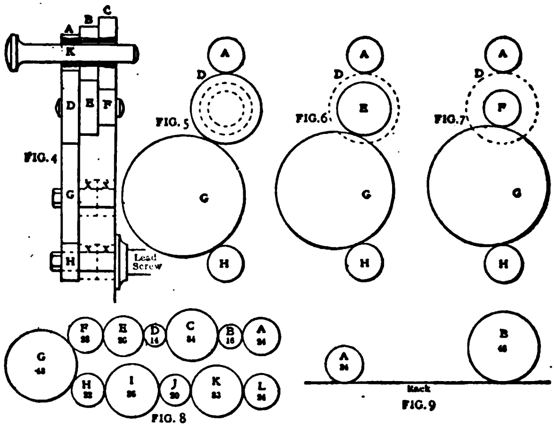
Tracing Out the Gear Movement

To find what thread will be cut with this arrangement, just pick out the gears which *drive* from the main spindle to the intermediate gear G. These are A and E, in Fig. 6, with 24 and 36 teeth. Multiply them together. This gives $36\times 24=864$. Now take the *driven* gears, D and H (G doesn't count at all), and multiply them together, getting $48\times 24=1152$. Multiply this by pitch of lead screw 6, and divide the whole thing by 864, which gives 8, showing an 8 thread will be cut.

The Gears to Use

Working this the other way, to find what gears to use to cut a 10 thread, we do it in almost the same way.

Multiply the two driving gears, *A* and *E*, together as before, and multiply this by the thread to be cut. This gives $24 \times 36 \times 10$, or 8640. Then multiply the driven gear *D*, 48 teeth, by the pitch of lead screw 6, and get $48 \times 6 = 288$.



Cone of Gears and Gear Train.

Divide 8640 by the 288 and find that 30 is the gear to put on the lead screw.

The Next Change

Shifting the gears *G* and *H* in toward the head and putting both collars *X X* on the outside, so *G* meshes into *F*, gives another step in the change of gears as shown in Fig. 7. As *D* has 48 teeth and *F* 24 teeth, *F* will drive *G* just half

as fast as D is being driven by A and gear the lead screw down just half, or make it the same as a 12 thread in simple gear.

Remember that gears B and C are running idle all this time and need not be considered at all.

This takes us through one set of changes and makes clear the process. Moving the pin in so that the key K engages B, we have an entirely different set of conditions; but they can be followed out in just the same way.

With gears G and H outside, as in Fig. 4, the gear B drives E, both having 36 teeth; D, with 48 teeth, drives G and H. The same calculations can be made as before, multiplying together the driving gears B and D and the thread to be cut and dividing this by the driven gear E multiplied by the pitch. This gives the thread that will be cut.

When Gears Drive Direct

No matter what the lathe or the gearing, when all the gears in the driving train are in direct line, as in Fig. 4, the gear H will turn just as many teeth as the number of teeth in A, for every revolution of the spindle; and with the gear H having the same number of teeth as the gear A, the thread cut will be the same as the lead screw.

To Cut 11½ *Thread*

This particular lathe is equipped with change gears from 24 to 66 varying by six teeth, and includes a 39-tooth gear, but none that will cut an 11½ thread, which could be done with a 69-toothed gear on the lead screw (which is 4 pitch) and the train in a direct line, as in Fig. 4. As 66 is the largest gear in the list, and this cuts an 11 thread, to

cut a 12 it is necessary to compound, which is done by leaving pin K in gear A and moving G in to mesh in F; a 36 is used on the lead screw. Just to prove this, and not take it for granted, figure it out. The driving gears are A and F, 24 teeth each. Driven gear is D, 48 teeth. $24 \times 24 \times 12 = 6912$. Multiply 48 by 4, the pitch of the lead screw, and get 192. Divide the first by the second and get 36 as the gear for the lead screw.

Cutting as Near as Possible

No regular combination of the gears gives $11\frac{1}{2}$ thread, but we can get very close to it by making a little compound gear of our own by taking off the intermediate gear G and fastening two other gears to run on that stud. With the gears in a single train, as shown in Fig. 4, it would take a 69-tooth gear to cut the $11\frac{1}{2}$ thread. But as these are not to be had, we must find something smaller, and make up for the difference by turning it a little more slowly. So we slip off the first collar X in the lead screw and fasten together a 42 and a 36 to place on G, with the larger gear meshing into D and the smaller into the lead-screw gear H. Trying this with a 66 gear on H, we have as driving gears 24 and 36, and as driven gears 66 and 42. Then $24 \times 36 = 864$, and $66 \times 42 = 2772$. Divide this by 864 and get 3.2 as the ratio between the lead screw and the thread to be cut. Multiply it by 4, the pitch of the lead screw, showing that this combination will cut a 12.8 pitch thread.

Cut and Try Method

This shows the lead screw must turn faster, so we try 60 on the lead screw instead of 66, and see what thread will

be cut. Repeating the multiplications, we get $60 \times 42 = 2520$. Divide this by the same number as before, as the driving gears have not been changed, and we get 2.91 as the ratio. Multiply this by 4 and find that it will cut a thread having 11.64 pitch, which will do for a short length of thread, if nothing better is at hand.

Trying one more combination, we use 48 and 42 as the compound gears, and come a little closer. The figuring in this case for the driving gear is $24 \times 42 = 1008$, and for the driven gears $60 \times 48 = 2880$. Dividing gives 2.85, and multiplying by 4 gives 11.4 threads per inch.

Another way of doing this, without decimals, is to multiply the driven gears by the pitch of the lead screw before dividing. This will give the thread that will be cut by the combination given. This would work out like this:

$$\frac{60 \times 48 \times 4}{24 \times 42} = \frac{11{,}520}{1008} = 11\tfrac{3}{7}, \text{ or } 11.43,$$

as you prefer. Figuring out various combinations in this way will show just what can be done, and by taking care to separate the driving gears from the driven and remembering to put the pitch of the lead screw with the driven gears, because it is driven with them, there will be no trouble even if a number of compounding sets are used.

Intermediate Gears Don't Count

A word of caution regarding intermediate gears may save mistakes in some cases.

Any gear which simply transmits motion from a driving gear to a driven gear can be left out of the question entirely. In Fig. 4, with the gears as shown, both *D* and *G* are inter-

mediate and have nothing to do with the thread cut. But moving *G* to mesh in *E* makes *D* a driven and *E* a driver, as both are fastened so as to move together.

In Fig. 8 is a train of gears of varying numbers of teeth, in which *A* is the driver and *L* the driven. Both *A* and *L* have 24 teeth. When *A* makes one turn, *L* must do the same, because when *A* moves one tooth, they all follow suit and *L* moves one tooth also, not allowing for lost motion.

In Fig. 9 the rack is practically an intermediate gear. With *A* the driver, *B* will move 24 teeth, or half-way around for every turn of *A;* and with *B* as the driver, *A* will move 48 teeth, or two revolutions, for every turn to *B*. So by leaving out all gears which simply transmit motion without changing its rate, and keeping driving gears in one lot and driven gears in another, there will be no trouble in figuring out what gears are needed for any pitch of thread.

Fractional Threads

The calculations for gears to cut the $11\frac{1}{2}$ thread show how any thread can be handled, but there is sometimes a chance for getting mixed up on threads that are odd and fast, such as $1\frac{1}{2}$ threads to 2 inches. This is $\frac{3}{4}$ of a thread to an inch and means that the lead screw must make four turns and move the carriage an inch while the spindle is making $\frac{3}{4}$ of a revolution. So the gear selected for the stud must be large enough to turn the lead screw fast enough to move the carriage $1\frac{1}{3}$ inches to each turn of the spindle. With a lead screw of 4 pitch this must turn four times $1\frac{1}{3}$ or $5\frac{1}{3}$ times as fast as the spindle. Using a 24-tooth gear on the screw would mean $5\frac{1}{3}$ times this, or a 128-tooth gear on the stud. If this is larger than you have at hand, use

a 64 and put in a 2 to 1 compound gear to double the speed of the intermediate gear, and the thread will be right.

Still Another Way

Sometimes you get an order to make a thread 1¼ inches pitch, meaning 1¼ inches between threads. The easiest way to handle this is to consider the pitch of the lead screw in the same way, as being ¼ inch between threads. Then as 1¼ inches is 5 times ¼ inch, the lead screw must turn five times as fast as the spindle, using a 120-gear on the stud, and a 24 on the lead screw, or a 60 on the stud, with a doubling-up compound gear in between.

If the thread is very odd, such as $\frac{57}{64}$ between threads, which isn't likely to happen, but which you want to be able to handle if it does, the same method holds good. As ¼ is $\frac{16}{64}$, the ratio is 16 and 57. Multiplying both by 2 gives 32 teeth for the lead-screw gear and 114 for the stud. Compounding 2 to 1 would give a 57-tooth gear, which is also odd, and 3 to 1 would give a 38, which is more apt to be on hand.

By carefully following each step you will have no trouble in cutting any kind of a thread wanted, even metric threads, which are sometimes called for.

Cutting Metric Threads

This can be done on any lathe by using a special pair of compound gears having 50 and 127 teeth. It may be necessary to make longer studs for the head and change gears in some cases.

With the 50-tooth gear as the driver of the pair, the 127-tooth being driven from the head, the carriage travel will

be reduced in the proportion of 2.54 to 1, because there are 2.54 centimeters to an inch.

The pitch of metric threads is given in millimeters, giving the distance in millimeters from one thread to the next. To use these "translating" gears, as the 50 and 127 are called, it is necessary to reduce the pitch to threads per centimeter, which is 10 millimeters. If the pitch is 2 millimeters, there will be five threads to the centimeter.

Then the lathe is geared just as though you were cutting five threads per inch, with a 24 on the stud and a 30 on the lead screw. The "translating" gears reduce carriage movement just in proportion as a centimeter is less than an inch.

Take care to get the thread measurement into the number of threads per centimeter, then it will be plain sailing.

Having a lathe which will cut a 4 thread with even gears on stud and screw, we wish to cut a 6 thread. What gears shall we use?

Multiply lead screw or 4 by any number, as 8, and get 32, the gear for the stud. Then multiply thread to be cut by the same number and get 48, as the gear for the lead screw.

With a 24-gear on the stud, what gear on the lead screw will cut a 20 thread?

Multiply stud gear 24 by thread to be cut, 20, and get 480. Divide this by pitch of lead screw and get 120 as gear for lead screw. This is probably larger than we have, so we must reduce the speed with a compound gear of say 2 to 1 or 3 to 1, letting the 24 gear on the stud drive a 48-gear and a 24 on the same shaft drive a 60 on the lead screw, just half the size that would be needed driving direct in a 2 to 1 compounding of gears.

To prove this, try it by the rule to find the pitch of the thread that will be cut with any gears. The pitch of lead screw multiplied by screw gear and any other driven gears is $4 \times 60 \times 48$, and this divided by stud and any other driving gear is $\frac{\cancel{4} \times \overset{10}{\cancel{60}} \times \overset{2}{\cancel{48}}}{\underset{\cancel{6}}{\cancel{24}} \times \cancel{24}} = 20$ as the thread that will be cut.

These rules will cover all the cases of thread cutting you will find and can be boiled down as shown below.

Having	To Find	Rule
True pitch of lead screw Thread to be cut Gear on the stud	Gear for lead screw	Multiply stud gear by thread to be cut and divide by true pitch of lead screw
True pitch of lead screw Gear or stud and any other driving gear in the train Gear on lead screw and any other driven gears in the train	Pitch of thread that will be cut	Multiply pitch of lead screw by screw gear and any other driven gears in the train, and divide by the stud gear and any other gears in the driving train
True pitch of lead screw and thread to be cut	Gears for stud and gear for lead screw	Multiply pitch of lead screw by any number as 4, 5, 6, or 8, that will give a gear in the train. This gear goes on the stud. Multiply thread to be cut by the *same number*, and the answer is number of teeth in gear to go on the lead screw

EXAMPLES

With a lead screw of 6 pitch, what gears can be used to cut a 7-pitch thread? *Ans.* 30 on stud, 35 on screw; 36 and 42; 48 and 56, etc.

What thread will be cut with a 40 gear on the stud and a 50 on the lead screw, if the lead screw is 4 pitch? *Ans.* 5-pitch screw.

With the lead screw 4 pitch, what gears are needed to cut a 11½-pitch-thread? *Ans.* 24 on stud, 69 on screw, or 32 on stud and 92 on screw.

With a 6-pitch lead screw, what gears will be needed to cut a thread having 1⅓ inches lead? *Ans.* 24 on the screw and 192 on stud, or a 64 on stud compounded 3 to 1.

To cut a thread of 2 millimeters pitch with lead screw 6 pitch, what gears are used with the translating gears of 50 and 127 on compound stud? *Ans.* 30 on stud, 36 on lead screw.

CHAPTER IX

SCREW THREAD CALCULATIONS

Pitch and Lead

BEFORE taking steps to cut any threads it is best to fix in the mind what the pitch of a screw is. As usually measured, we say 10 pitch, meaning 10 threads to the inch, or 20 or 9, or any other number. On the other hand, we sometimes run across a drawing marked $\frac{3}{4}$ pitch, which should mean three quarters of a turn to the inch or one turn in $1\frac{1}{3}$ inches. If it says $\frac{3}{4}$-inch pitch it means $\frac{3}{4}$ inch from one thread to the next.

The next point to watch is "pitch" and "lead." The pitch of a thread is the distance from the center of one tooth to the center of the next. The lead of a screw is the distance a nut will advance in one revolution of the screw. If it is a single-thread screw, the pitch and lead are always the same; but for double, triple, or any multiple thread, the lead is just as many times the pitch as there are multiple threads. A double thread has a lead twice the pitch, a triple screw three times, and so on. The pitch might measure 12 with a thread gage, but be a quadruple thread of 3 to the inch. The angle of the thread around the bar tells the story here.

The United States Standard Thread

The United States Standard or Franklin Institute or Sellers thread is so called because it was designed by Wm. Sellers, recommended by the Franklin Institute, and has been adopted by the United States Government. This thread has the same angles as the V, 60 degrees, but has one eighth of the depth taken off the top and bottom, as shown in Fig. 1. This gives the proportions for a thread of 1 pitch or 1 inch from one to the next.

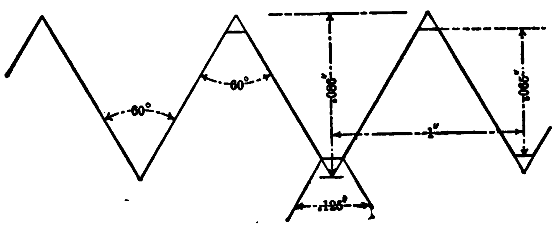


FIG. 1. — United States Standard Thread.

The depth of a V-thread of this pitch is 0.86 inch, and of the United States Standard 0.65 inch, while the flat at top and bottom is 0.125 inch. To find the depth of any other thread, divide these figures by the number of threads to the inch. To help in allowing for the thread when boring a die or other piece with internal thread, Table 1 will be found useful. This also gives the width of the flat for the point of the thread tool, but it is fully as easy to measure this with a standard thread gage and there is much less chance of error. Simply grind the tool to fit the gage for whatever thread is to be cut, being sure it is a United States Standard thread gage and not a V.

TABLE 1.—PROPORTIONS OF THREADS

Pitch or Threads per Inch	V. Threads		U. S. S. Threads		
	Depth of Thread on One Side	Depth of Thread on Both Sides	Depth of Thread on One Side	Depth of Thread on Both Sides	Width of Flat at Top and Bottom
1	0.86	1.72	0.65	1.30	0.125
2	0.43	0.86	0.325	0.65	0.0625
3	0.29	0.58	0.22	0.44	0.042
4	0.216	0.43	0.16	0.33	0.031
5	0.17	0.35	0.13	0.26	0.025
6	0.14	0.29	0.11	0.22	0.021
7	0.12	0.25	0.093	0.19	0.018
8	0.11	0.22	0.08	0.17	0.016
9	0.095	0.19	0.072	0.15	0.014
10	0.086	0.172	0.065	0.13	0.0125
11	0.08	0.16	0.06	0.12	0.011
12	0.07	0.15	0.055	0.11	0.01
13	0.075	0.15	0.05	0.10	0.009
14	0.06	0.13	0.046	0.092	0.009
16	0.055	0.11	0.04	0.08	0.008
18	0.05	0.10	0.036	0.072	0.007
20	0.043	0.09	0.033	0.066	0.006

Cutting Double Threads

A single thread is simply one continuous turn around a bar, called a helix by the professors, but it can be thought about easier if we think of a spring wrapped around a mandrel, as in Fig. 2, where it makes a round thread.

If we wanted a faster thread, we could stretch the spring

out as in Fig. 3, leaving gaps in between as shown. If these were threads, we might call the first 4 to the inch and the latter 2 to the inch, as in Figs. 4 and 5. In Fig. 5 we have cut away all the metal between the threads to make it resemble the spring more closely, but this would be a hard thread to cut and is not desirable. What should really be done is to cut the thread as in Fig. 6, leaving the metal in between, and then cutting a second thread in between the others just as though we wound a second spring in between the coils of the first in Fig. 3.

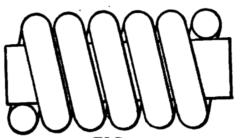


FIG. 2

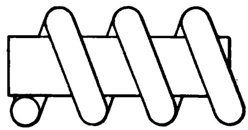


FIG. 3

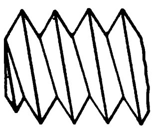


FIG. 4

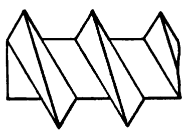


FIG. 5

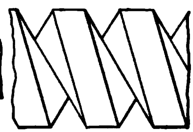


FIG. 6

Single and Double Threads.

The reason for the faster thread of 2 to the inch is to get a half-inch movement for every turn of the screw instead of one-quarter inch with the 4 thread.

If the screw is large enough in diameter to stand the depth of a 2 thread, then there is no need of a double thread; but take a screw as in Fig. 7, $1\frac{3}{8}$ inches in diameter, and see how a single thread of 2 to the inch weakens the screw. Fig. 8 shows the same diameter screw with two threads of

2 pitch cut one half their total depth or the same depth as a 4 thread.

Having decided on the double thread to be cut, the first thread is cut to one half the depth for that pitch and the second thread cut half-way between the grooves of the first

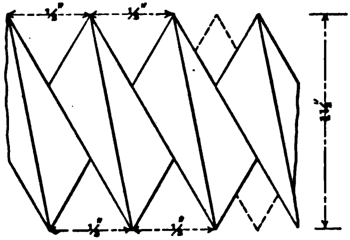


FIG. 7

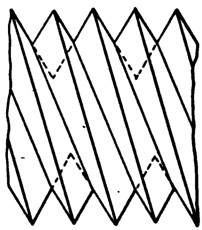


FIG. 8

Single and Double Threads

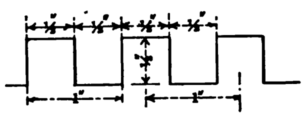


FIG. 9

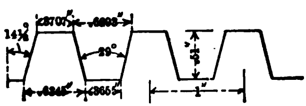


FIG. 10

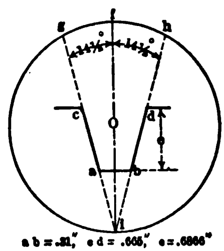


FIG. 11

Square, Acme and Worm Threads.

thread. This space can be divided by measurement, by turning the work half-way round or by turning the *stud gear* just half-way round. The last is probably the easiest except when an odd-tooth gear, as when the change gears jump by 5, is on the stud. In that case it is probably easier to measure and reset the tool.

Square Threads

Square threads are sometimes puzzling in two ways: in grinding the tools and in cutting the thread. The width of the tool is half the pitch because the land and the space must both be the same, although in practice it is necessary to make the space a little wider to allow for play and for any variation in the pitch. In Fig. 9 a pitch of 1 to the inch is shown. This means either the distance from center to center of threads, or from the face of one thread to the corresponding face in the next thread. If it is a double or triple thread, we must be careful and not confuse pitch and lead. The depth of the square thread is usually the same as the width of the land or the space, although here again there is a difference of opinion, some allowing clearance at the bottom. The square thread is rather difficult

TABLE 2.—SQUARE-THREAD TOOLS—WIDTH AND DEPTH OF THREADS ARE THE SAME

	Width of Thread Tool		
Pitch or Threads per Inch	Single Thread	Double Thread	Triple Thread
1	0.5	0.25	0.166
2	0.25	0.125	0.083
3	0.166	0.083	0.055
4	0.125	0.062	0.042
5	0.10	0.05	0.033
6	0.083	0.042	0.028
7	0.071	0.035	0.023
8	0.062	0.031	0.021

to cut on account of giving clearance on the sides to avoid rubbing from the angularity of the thread.

In cutting double threads of square section, the same precautions must be observed as with V or other threads. Table 2 gives the width of square-thread tools for use in cutting single, double, and triple threads.

Top rake on square-thread tools gives a good cutting edge, and the chips can often be rolled out in a hurry if the stock is good and clear.

Acme Threads

Square threads were not always easy to cut, and so it often happened that feed screws, lead screws, etc., were made flat top and bottom, but with slanting sides of any angle that pleased the eye of the man who ground the thread

TABLE 3.—PROPORTIONS OF ACME THREADS

No. Threads per In. Linear	Depth of Thread	Width at Top of Thread	Space at Bottom of Thread	Space at Top of Thread	Thickness at Root of Thread
1	0.5100	0.3707	0.3655	0.6293	0.6345
2	0.2600	0.1853	0.1801	0.3147	0.3199
3	0.1767	0.1235	0.1183	0.2098	0.2150
4	0.1350	0.0927	0.0875	0.1573	0.1625
5	0.1100	0.0741	0.0689	0.1259	0.1311
6	0.0933	0.0618	0.0566	0.1049	0.1101
7	0.0814	0.0529	0.0478	0.0899	0.0951
8	0.0725	0.0463	0.0411	0.0787	0.0839
9	0.0655	0.0413	0.0361	0.0699	0.0751
10	0.0600	0.0371	0.0319	0.0629	0.0681

tool. As these were neither square nor V, they soon had a name of their own and were called bastard. In some parts of the country this term is applied only to odd pitches, but any old hand will recall bastard threads of a great variety of shapes and sizes. Of course no two of these were alike, and the natural course of events brought about a standard which is now known as the Acme thread. The proportions for a pitch of one to the inch are shown in Fig. 10, and Table 3 gives full details for other sizes. Thread gages can be had for the Acme thread if desired.

Worm Threads

The Acme thread is so near the worm thread that care must be taken to avoid using one for the other or getting the proportions mixed. The angle is the same, 29 degrees;

TABLE 4.—BROWN & SHARPE WORM THREAD PROPORTIONS

WORM THREADS

Pitch in Threads per Inch	Lead per Revolution	Depth of Thread	Width of Tool Point or Bottom of Thread	Width of Top of Thread	Width of Space at Top of Thread	Width of Root of Thread
1	1	0.6866	0.31	0.335	0.665	0.69
2	0.5	0.3433	0.155	0.167	0.332	0.345
3	0.333	0.2288	0.103	0.111	0.222	0.23
4	0.25	0.1716	0.077	0.084	0.166	0.17
5	0.20	0.1373	0.06	0.067	0.133	0.14
6	0.166	0.1144	0.05	0.056	0.111	0.115
7	0.141	0.0981	0.044	0.048	0.095	0.098
8	0.125	0.0858	0.039	0.042	0.085	0.086

but the depth is greater, as can be seen in Fig. 11. This also shows an easy way to lay out the angle of 29 degrees if you want to make a gage for yourself.

Take a piece of sheet iron, draw a circle say 2 inches in diameter; draw a line through the center, as *f i*. Take one quarter the diameter, or ½ inch, in the dividers and mark off *g* and *h* from *f*. Connect *g i* and *h i* and the enclosed angle is 29 degrees.

The point of the tool *a b* is 0.31 × the pitch: the space *c d* is 0.665 × the pitch; and the hight *e* is 0.6866 × the pitch, according to the Brown & Sharpe standard, or practically one third deeper than the Acme thread. The details are given in Table 4.

EXAMPLES

What is the lead of an 8-pitch single-thread screw? *Ans.* 0.125 inch.

If a thread makes one turn in 1½ inches, what lead is it? *Ans.* ⅔ of an inch.

With a double thread, measuring ¼ inch between the points, what is the lead? *Ans.* ½ inch.

What is the difference between the Acme and the Worm thread? *Ans.* Angle is same, worm thread is deeper.

What is the width of a square thread tool for a 2-pitch double thread? *Ans.* ⅛ inch.

CHAPTER X

DRILLING FOR TAPS

Before giving a rule for this it is better to make a few sketches on paper or metal so there can be no doubt as to just what we are doing. Both the V-thread and the U. S. S. (or Sellers or Franklin Institute) thread have sides 60 degrees from the center line of the screw, so the first step is to lay off a thread with a large pitch, say one inch from

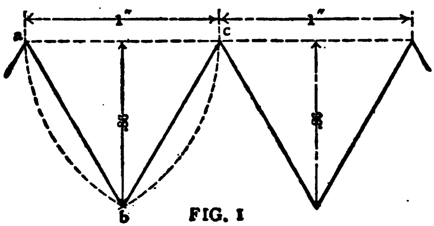


FIG. 1
Sharp V-Thread.

point to point, as in Fig. 1. There is no need for a 60-degree triangle to do this. Take one inch in a pair of dividers and, starting from *a*, draw the arc *b c;* then, with the point on *c*, draw *a b;* join *a b* and *b c*, and the thread is done. Continue this as shown for further use.

Now measure the distance from the line at the top of threads to the bottom, which will give the depth of a thread of one-inch pitch, or one to the inch. This will be found to be 0.866 of an inch, and in most cases you can call it

0.86 with perfect safety. This shows the depth of one thread, one to the inch, to be 0.86 inch, so that the threads on both sides of a bolt or other piece of work would be double this, or 1.73 inch, allowing for the last 0.006 that was dropped before.

If we want to know the depth of both threads on a bolt having 10 to the inch, we simply divide 1.73 by 10 and get 0.173 as the difference between the outside diameter of the tap and the diameter of the drill to use. If this is on a $\frac{3}{4}$-inch bolt, the correct size of hole will be $0.75 - 0.173 = 0.577$, or a little over $\frac{9}{16}$ inch. The same allowance should be made, no matter what the diameter of the work is, as long as the thread is 10 to the inch.

The rule, then, for V-threads is to divide 1.73 by the number of threads per inch, and subtract this from the outside diameter of the thread to be tapped or chased.

In most cases it will be better not to allow the full depth of the thread for tapping or chasing, except perhaps in brass, where a sharp thread is desired. In tapping cast iron it is customary to drill large enough so that the thread will not be quite full, owing to the tendency to crumble if left sharp; and in the case of wrought iron or soft steel, the metal is apt to crowd up in the thread, so that a scant allowance will give a full thread.

The depths of U. S. S. threads are found in the same way, but the figures are different.

Continuing the threads as in Fig. 2, we have the same angles, but $\frac{1}{8}$ the depth is taken from the top of the thread and filled in at the bottom, leaving a flat top and bottom and making the thread $\frac{3}{4}$ as deep as the V-thread. So, instead of being 0.866 deep, it measures practically 0.65

inch deep. Doubling this to take in both sides, we can use the same rule as before by simply changing 1.73 to 1.3 and figuring as before.

In using this for taps it is safe to take the nearest drill *larger* than the size given by the figures, unless the thread is very fine indeed.

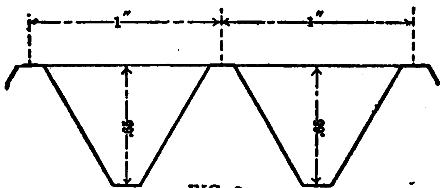


FIG. 2

A rough and ready way which does not give a full thread is to subtract one part of the threads per inch from the diameter. So for an inch tap with an 8 thread we would say 1 inch minus $\frac{1}{8} = \frac{7}{8}$ inch for the hole, leaving $\frac{1}{16}$ inch on a side for the thread.

EXAMPLES

What is the depth of a V-thread of 2 pitch? *Ans.* 0.43 inch.

Of a United States Standard thread? *Ans.* .325 inch.

What must the bore of a hole be to chase for a 2-inch bolt with a 6-pitch, V-thread? *Ans.* 1.712 inch.

What is the diameter of a 1-inch 8 thread U. S. S. thread bolt at the bottom of the thread? *Ans.* .84 inch.

If we bore a hole to 1.812 inches for a 2-inch V-thread tap of 6 pitch, how near a full thread will it be? *Ans.* This allows .094 inch on each side, instead of .144 inch as required for a full thread.

CHAPTER XI

TAPER WORK

Any machinist who has tried to turn a piece of work perfectly straight knows the difficulty of getting the tail center set just right to give exactly the same diameter at each end. But when it comes to setting the lathe for just the right taper, especially without useless trials that take a lot of time, it isn't any easier than turning a piece straight.

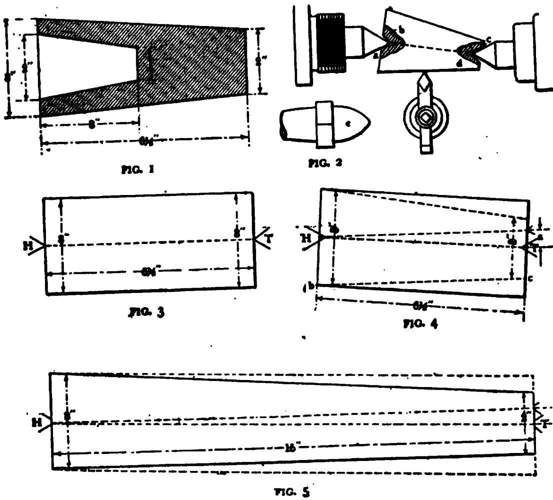
Next to selecting the change gears for thread cutting, the question of how much to set the tailstock over for a given taper is asked more often than any other. While it is a simpler problem than the other, in some ways it has some points which are not always clear.

Measuring the Taper

The first thing to consider is just what the taper is, as this is sometimes a point for differences of opinion. Some measure the taper on each side, while the usual way is to take the total taper, as with pipe threads. The standard pipe taper is $\frac{3}{4}$ inch to the foot, which is the same as $\frac{1}{16}$ inch to the inch, or 1 inch in 16 inches, whichever way you like best. In many cases it is easier to have the taper per inch, but we have to take these things as they come to us on the blue-prints or drawings.

The amount of taper depends on the diameter at the

two ends of the taper part of the bar and on the length of the taper, but the amount of offset for the tailstock depends on the length of the whole bar, regardless of how much is turned taper. In Fig. 1 is a section of a taper plug with a taper hole. The difference in diameter is 1 inch in both



Turning Taper Work.

cases, but the taper of the hole is much sharper, as it is less than half as long as the plug. The outside is 1 inch taper in 6½ inches, while the hole is 1 inch in 3 inches.

The Lathe Centers

There is some question as to what the effect of the depth the centers enter the work has on the setting over of the tailstock; but as it is practically out of the question to set

it over the exact distance the first time in any case, this need not enter into the question for general work. Fig. 2 shows a very short piece being turned at a sharp taper. It can be seen how the outer edge *a* and the point *b* bear on one end; on the other it is opposite, as in *c* and *d*. This is very hard on centers and makes it difficult to keep them from cutting. In fact, if much taper work is to be turned, a point more blunt than the usual 60 degrees may be found better. Some use a somewhat rounded point, as in *e*, Fig. 2.

Forgetting about the center and its action, and considering that the work is held between the points of the centers as in Fig. 3, *H* being the head and *T* the tail center, how much must we set the tailstock *T* over to cut the outside taper shown in Fig. 1? This is 3 inches at one end and 2 inches at the other, so we must reduce the small diameter 1 inch, which means ½ inch on a side, and we set the tailstock over ½ inch, as at *a* in Fig. 4. The tool moves along the line *b c* and cuts off ½ inch at the small end, running out at *b*. If the tailstock has been set over just ½ inch, the outside will be the correct taper, as shown by dotted lines.

In Fig. 5 the tailstock is set over the same amount and the small end reduced to 2 inches as before, but as the piece is 15 inches long instead of 6½ inches, the taper per inch or per foot is very much less.

Count Full Distance Between Centers

Taking the case of a bar 3 inches in diameter and 30 inches long, how much must it be set over to make a taper of 2 inches in 24, or within 6 inches of the whole length?

As the taper is 2 inches in 24 or 1 in 12 inches, or $\frac{1}{12}$ of

an inch in 1 inch, in 30 inches it would be $\frac{30}{12}$ or $2\frac{1}{2}$ inches; so the tail center must be set over one half of this or $1\frac{1}{4}$ inches. An example of this work is in turning the taper on the end of a piston rod, where the taper may be 6 inches long and perhaps an inch to the foot, as in the above case. The rod may be 48 inches long, and the whole length must be considered in setting over the tail center. In 48 inches the taper would be $\frac{48}{12}$ or 4 inches, so that the tailstock would have to be set over 2 inches. The two points to remember are the getting of the right taper and always to consider the total length of the piece regardless of the taper portion. And it makes no difference where the taper portion is, whether at the tail end, the middle, or near the headstock, the set-over is the same in any case.

Ways of Figuring Tapers

It is generally easier to figure tapers if we reduce them to the amount per inch in order to get at the offset. If the taper is given per foot, we divide it by 12, as, if it is pipe-thread taper of $\frac{3}{4}$ inch per foot, we have $\frac{1}{12}$ of $\frac{3}{4}$, or $\frac{3}{48}$ or $\frac{1}{16}$ per inch. If the taper is given 1 in 8 or 1 in 15, the taper per inch is of course $\frac{1}{8}$ or $\frac{1}{15}$ inch to the inch.

Tapers are frequently given in degrees, and in such cases they are usually turned with a compound rest divided into degrees; but it is sometimes handy to know what the taper would be in inches, so we give a table which may help if you get a blue-print with the taper marked in degrees. It is given to four places of decimals and you can use as many as circumstances seem to demand.

Tapers in Degrees

This table shows that to cut a taper of 4 degrees on a bar 10 inches long means a total taper of 0.708 inch, and that the tail center must be set over one half of this or practically 0.4 inch.

The table will also be a guide in the opposite direction by giving something of an idea as to what angle a given taper is. Taking the pipe taper of ¾ inch to the foot, or $\frac{1}{16}$ inch to the inch, what angle is it? Consulting a table of decimal equivalents, or dividing 1 by 16, gives 0.0625, and this is between 3 and 4, very nearly 3½ degrees.

In the same way a taper of 1 in 12 or $\frac{1}{12}$ inch to the inch is 0.083, which comes very close to being 5 degrees; to cut this, we set the tailstock over 0.042 inch for every inch of length in the bar.

TAPERS IN DEGREES AND THE EQUIVALENTS IN INCHES

1	0.0175
2	0.0349
3	0.0524
4	0.0708
5	0.0873
7½	0.1310
10	0.175
15	0.2633
20	0.3526
25	0.4434
30	0.5360
35	0.6306
40	0.7279
45	0.8284
50	0.9326
55	1.0411
60	1.1547

Points to Remember

The main points about taper work are:

Always consider the length of the work or the distance between centers instead of the length of the taper portion.

Make the offset half the amount of the total taper, which would be the difference between the diameters of the large and small ends if the taper were extended the whole length of the piece.

Always set the point of the tool at the hight of the lathe center in taper work. If it is set above or below the center, there will be a slight change in the taper between each cut, or as the diameter is reduced. You can see this by imagining the tool set ½ inch below the center. When the work gets down to 1 inch in diameter, the tool will not cut at all at the small end, but will cut more and more as it feeds on to the larger diameter. This is a very important point to remember.

In cutting threads, set the tool square with the center line of the work and not square with its surface.

It is always well not to set the tailstock over the full amount at first, so that the small end will be larger than required, rather than smaller. If the taper is too sharp, from setting the tailstock over too much, there is danger of spoiling the piece by having the taper run up too far on the bar in making a fit into a tapered hole. In making long taper fits it is easier to have the bearing for a short distance on each end with a relief in the center, and this generally answers the purpose equally well.

EXAMPLES

How much should the tailstock be set over for a taper bar 18 inches long to be $\frac{1}{2}$ inch to the foot? *Ans.* $\frac{3}{8}$ inch.

What is the standard pipe thread taper? *Ans.* $\frac{3}{4}$ inch per foot, 1 inch in 16 or $\frac{1}{16}$ inch in 1 inch.

How much should the tailstock be set over to turn a taper 6 inches long on a 12-inch bar, the taper to be $1\frac{1}{2}$ inches at small end and 2 inches at the large end? *Ans.* $\frac{1}{2}$ inch.

How many degrees from the center line of the cross slide would you set the compound rest to bore a cone having 70 degrees included angle? *Ans.* 55 degrees.

CHAPTER XII

SPEED OF LATHES, PLANERS, AND SHAPERS

THE cutting speed of a lathe tool depends on the diameter of the work and the number of revolutions that it makes per minute, so that to find how fast a shaft or other piece of work must turn to give, say, 60 feet cutting speed, we first find the diameter, which call 10 inches. The distance around the piece will be $10 \times 3.1416 = 31.416$ inches or dividing by $12 = 2.618$ feet. Divide 60 by this and we get, $60 \div 2.618 = 22.9$, practically 23 turns a minute.

How long will it take to turn 3 feet of this with a feed of $\frac{1}{16}$ inch per revolution?

This depends entirely on the number of revolutions per minute, and not on either the diameter or cutting speed. With 23 turns a minute and $\frac{1}{16}$-inch feed per turn, gives $\frac{23}{16}$ or $1\frac{7}{16}$ inches feed in one minute. So we find the number of times $1\frac{7}{16}$ inches goes in 3 feet or 36 inches, and we have the time required for the cut, not allowing for any delay in sharpening tools, etc. Dividing 36 inches by $1\frac{7}{16}$ gives $25\frac{1}{23}$ or practically 25 minutes for the cut. Increasing the feed to $\frac{1}{8}$ inch per revolution would cut the time in half, if the work and the tool can stand it.

The time of milled work can be figured in exactly the same way if the feed is given in parts of an inch per revolution of the cutter; if, however, the feed is given in inches

per minute, as is frequently the case in milling machine or surface grinding work, the revolutions of the cutter do not enter into the calculation at all, although it must run at its proper cutting speed to allow of such a feed to the work.

A table of cutting speeds will sometimes be found handy either for lathe work or milling cutter, but it is better to be independent of all tables, and the rules given on page 74 are simple and easily learned.

This will enable you to estimate very closely on work of this kind.

EXAMPLES

With a cutting speed of 40 feet a minute, how long will it take to turn a 6-inch bar 30 inches long, with a $\frac{1}{8}$-inch feed? *Ans.* A little under 10 minutes.

At 60 revolutions per minute, how long will it take to make a roughing cut with $\frac{3}{16}$-inch feed, and a finishing cut with $\frac{1}{16}$ feed — both cuts 21 inches long and allowing one minute for changing tools? *Ans.* 8.46 minutes.

What feed is necessary to run a cut of 32 inches in 8 minutes at 40 revolutions per minute? *Ans.* $\frac{1}{10}$ of an inch feed per revolution.

With a feed of $\frac{1}{8}$ inch per revolution, how fast is it necessary to run a bar, to turn 40 inches long in 10 minutes? *Ans.* 32 revolutions per minute.

ACTUAL CUTTING SPEED OF PLANERS

If a planer has a forward movement of 20 feet a minute and the return is at the same speed, it is very clear that the actual cutting speed is one half of 20 or 10 feet, as it spends half the time in going back for a new cut. Calling the cut 20 feet long, it would take it one minute to go forward and

Rules for Cutting Speeds

Having	To Find	Rule
Diameter of work or milling cutter and cutting speed desired	Revolutions per minute necessary	Multiply diameter of work by 3.1416 and divide desired cutting speed by it.
Diameter of work or milling cutter and revolutions per minute	Cutting speed	Multiply diameter by 3.1416 and by revolutions per minute.
Revolutions of machine per minute and desired cutting speed	Diameter of work or milling cutter	Divide desired cutting speed by revolutions per minute and divide this by 3.1416.

Rules for Time Required

Having	To Find	Rule
Revolutions per minute Feed per revolution Length of cut	Time required	Divide length of cut *in inches* by feed per revolution in parts of an inch and by revolutions per minute, or multiply feed by revolutions and divide length of cut by this.
Revolutions per minute Length of cut Time required	Feed per revolution	Multiply revolutions per minute by time required and divide length of cut by this.
Length of cut Time required Feed per revolution	Revolutions per minute	Multiply feed per revolution by time required and divide length of cut by this.
Time required Feed per revolution Revolutions per minute	Length of cut	Multiply time required by feed per revolution and by revolutions per minute.

another minute to go back, so that while the table travels 40 feet in 2 minutes it is only cutting during 20 feet of the distance, and as the round trip takes 2 minutes, the actual cutting speed per minute is only 10 feet.

Increase the return speed to 40 feet, and see what happens. Now the forward cut of 20 feet takes one minute as before, but the return stroke is made in half the time, so that the round trip takes 1½ minutes, or the 20-foot cut takes 1½ minutes instead of 2. As we cut 20 feet in 1½ minutes, in one minute we cut ⅔ of 20 feet or 13⅓ feet actual cutting speed; so that doubling the return speed has only increased the actual cutting rate 3⅓ feet a minute.

Now going to the extreme of a return five times as fast as the cutting speed, or 100 feet per minute, and we still have the 20-foot cut made in one minute, but the return stroke only takes ⅕ of a minute, so that the round trip takes 1⅕ minutes. Dividing 20 by 1⅕ gives 18⅓ feet per minute for actual cutting speed, so that in spite of the high return speed the actual cutting speed is not doubled. In fact it would be impossible to double it as it would be necessary to return without consuming any time whatever, and this is out of the question.

By increasing the cutting speed to 25 feet a minute, and using a return of 3 to 1, we have the 25-foot cutting stroke in 1⅓ minutes. Dividing 25 by 1⅓ we have 18¾ feet actual cutting speed for the planer. This shows that it helps more to increase the forward speed by 25 per cent than to increase the return speed by 500 per cent.

It is also easier on the mechanism of the planer as the shock of reversal is much less with a moderate return speed.

Cutting Speed of Shapers

When it comes to shapers we have an entirely different proposition, for here we count by revolutions per minute and not by cutting speed.

Just suppose the shaper is running 30 revolutions, and making a cut one foot long at each stroke. Then it is cutting 30 feet of metal per minute, regardless of the return speed, but it is cutting it at a faster rate than this, depending on the return speed ratio.

If it returns at the same speed as the cutting stroke, then as both strokes occur 30 times a minute, each takes half the time and goes at twice the speed, or 60 feet per minute.

If it returns at twice the cutting speed, then the return stroke takes $\frac{1}{3}$ and the cutting stroke $\frac{2}{3}$ of the time. As the tool travels 60 feet a minute, counting both forward and backward strokes, and it makes 30 backward strokes or 30 feet in $\frac{1}{3}$ of a minute, it travels $3 \times 30 = 90$ feet a minute on the return stroke. And as it makes 30 forward strokes or 30 feet in $\frac{2}{3}$ of a minute, then in one minute it travels at the rate of $1\frac{1}{2} \times 30 = 45$ feet, one half as fast as the return stroke.

This is entirely different from the planer, as in this case the greater the return speed the slower the cutting speed for the same number of revolutions. But this does not mean that there is no object in having a quick return, for it allows the shaper to be speeded up enough faster to make the cutting speed as fast as with the slower return.

EXAMPLES

What is the actual cutting speed of a planer having 20 feet forward and 30 feet return per minute? *Ans.* 12 feet per minute.

What increase is made by increasing the return to 60 feet per minute? *Ans.* 3 feet, from 12 to 15.

Instead of increasing the return, suppose we increase the cutting speed to 30 feet and have the return 40 feet, what is the actual cutting speed? *Ans.* 17.1 feet.

With a forward speed of 30 feet per minute, what return will be necessary to get an effective cutting speed of 20 feet? *Ans.* 60 feet return speed.

CHAPTER XIII

SQUARE AND CUBE ROOT

WE know that if we have a square frame with 12 holes up and down, and 12 holes across, we can find the total number by multiplying 12 by 12 = 144 holes in all, the same as the area of a 12-inch square in square inches. In other words, multiplying a number by itself is called squaring it, which is easy, but reversing the process to find the number of holes on a side of a square to contain 144 holes is a little different proposition, and is called finding the square root. This is needed in many shop calculations, and is not difficult if we go at it right. The rules are easy, but it is better to look into the reasons a little before we learn the rule.

One of the common uses is in finding the distance across the corners of a square: such as to find the long diameter of a square nut. Fig. 1 shows that three squares with one side equal to the sides of a right-angled triangle are so proportioned that the square on the long side or hypothenuse is always equal to the other two. In other words, if the sides of a triangle are 3, 4, and 5 inches, we know that the square of 3 plus the square of 4 will equal the square of the other side, because we have $3 \times 3 = 9$, $4 \times 4 = 16$, and $9 + 16 = 25$, which is the square of 5.

Now suppose we have a right-angled triangle with one

of the square sides 9 inches and the other square side 12 inches, it is plain that the third or slant side or hypothenuse is such a length as, multiplied by itself or squared, will equal the square of 9, or 81, plus the square of 12, or 144,

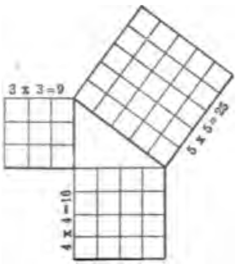


FIG. 1

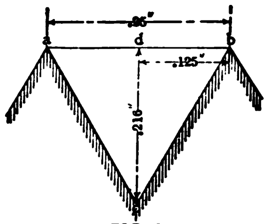


FIG. 2

Square Root Examples.

which is 225. This brings us to the necessity for the rule for finding what number, multiplied by itself, equals 225, or the rule for square root.

Rule for Square Root

Divide the number into periods of 2 figures each, beginning at the right, by making dots over the first, third, fifth, etc., figure, as $\dot{2}2\dot{5}$, beginning at the 5. Find the largest number which, when squared, will go into the first left-hand period or $\dot{2}$. This number will be the first part of the root. The largest number is 1, as $2 \times 2 = 4$, and this will not go in 2 so we put 1 in the answer at the right. Square this and subtract from the 2, leaving 1. Bring down the next period, making it 125. Double the root already found, making 2 for the trial divisor into the first two figures of the new dividend, and we see that 2 in 12 will go

no more than 5 times, as this is only a trial division with part of the divisor, so we put 5 in the answer and also after the trial divisor, making the final or true divisor 25. Then $5 \times 25 = 125$, which just goes into the dividend, so that we know the square root of 225 is 15, and can prove it by multiplying 15 by 15 and getting 225. The whole operation is:

Number pointed off $= \dot{2}2\dot{5}(15$ *Ans.*

	1
$2 \times 1 = 2 =$ trial divisor.	125 number brought down =
2 in 12 = 5, so put 5 after 2, making 25 the final divisor.	
$5 \times 25 =$	125
	No remainder.

Taking another example, find the square root of 189225.

Begin at right and point off		$\dot{1}8\dot{9}2\dot{2}5(435$
Largest square in 18 is 4.	$4 \times 4 =$	16
		292
Doubling 4 = trial divisor =	8	
8 in 29 = 3.		
Put 3 after 8 = final divisor	83	
$3 \times 83 =$		249
Doubling answer for trial divisor = 86		4325
86 in 432 = 5		
Put 5 after 83 for final divisor = 865		4325

Decimals are handled in just the same way, except in pointing off we put the first mark over the second figure

after the decimal point, so the periods will have two figures just the same, as square root of $\dot{1}4\dot{4}.4\dot{9} = 12.07$.

Instead of writing "square root" we use the sign $\sqrt{\ }$ and, if this is only to include part of the numbers, we use a bar or vinculum to show how many, as: $\sqrt{3 \times 4 \times 5} + 16$ means that only the square root of the numbers under the bar is wanted; then the 16 is added to the answer.

Suppose we have a 10-inch square and want to find the distance across the corners. We know that the square of the diagonal equals the square of two sides added together, so we have $10 \times 10 = 100 \times 2 = 200$, and the $\sqrt{200} = 14.14$ inches.

In the same way, if we have any right-angled triangle and want to find the hypothenuse or slant side, we know that if we find the sum of the squares of the other sides and take the square root, we have the slant side and we can say:

The slant side = $\sqrt{\text{sum of square of other two sides}}$, and either of the other sides =

$$\sqrt{\text{square of slant side} - \text{square of other side}},$$

as can be seen from Fig. 1.

Another common case is finding the depth of a V-thread. Here we have the distance from one thread to the next as the base of an equilateral or equal-sided triangle, and as the depth is the distance from the point to the bottom, we divide the triangle, as in Fig. 2, to make it a right angle. We now have the end *d b*, and the slant side, *b c*, to find the other side, *d c*. Calling the thread 4 to the inch the lead will be .25, so that the distance *d b* will be .125, and *b c* will equal .25. Then the distance $d\,b = \sqrt{b\,c^2 - d\,b^2}$. $.25 \times .25 = .0625$, and $.125 \times .125 = .015625$. Then $.0625 - .015625 = .046875$. The square root of this equals the depth *d b*, and is

		$.\dot{0}4\dot{6}8\dot{7}5(216$
	$2 \times 2 =$	$\underline{4}$
Trial divisor $2 \times 2 =$	4	68
4 in 6 = 1		$\underline{41}$
Final divisor = 41		2775
$1 \times 41 = 41$		
$2 \times 21 =$ trial divisor 42		
42 in 277 = 6		
Final divisor =	426	
$6 \times 426 = 2556$		$\underline{2556}$
		219

Point off as many places in the answer as there are periods in the dividend, making the answer .216 with a remainder. If it was necessary to carry it further we would add two ciphers to the remainder and go ahead. So we know the depth of a 4-pitch V-thread is .216 inches.

By using a good table of squares much square root calculation can be avoided, but it is well to know how to do it when you want to.

EXAMPLES

Find the square root of 8281. *Ans.* 91.

Find the square root of 11881. *Ans.* 109.

Find the square root of 211. *Ans.* 14.525 +.

What is the hypothenuse of a right-angled triangle having sides 12 and 16 inches long? *Ans.* 20 inches.

If a right triangle is 18 inches on the base and 30 inches on the slant side, what is the other side? *Ans.* 24 inches.

FINDING THE CUBE ROOT

CUBE root is an enlargement on square root, and is the method of finding what number multiplied by itself twice will equal a given number. It is not used nearly as often as square root, but it is well to know how it is done.

Point off the number 12167 in three places, instead of two, as $1\dot{2}16\dot{7}$, and we know that the largest number that will go in 12 is 2, so this goes in the answer, and we bring down the rest, 4167. Square the root already found, multiply it by 3 and annex two ciphers. See how many times this goes into 4167. Squaring $2 = 2 \times 2 = 4$, and $4 \times 3 = 12$. Then 12 in $41 = 3$, so this is the next figure in the answer. Add three times the product of the last root figure by the rest of the root and annex one cipher to it, then add the square of the last root figure. This gives the true divisor and must be multiplied by the last figure in the answer.

To find the cube root of 12167 point off

		$1\dot{2}16\dot{7}(23$
$2 \times 2 \times 2 =$		8
		4167
$2 \times 2 \times 3 = 12$		4167
Annex two ciphers,		
making trial divisor	1200	
$3 \times 3 \times 2 = 18$		
Annex one cipher =	180	
$3 \times 3 =$	9	
Final divisor	1389	

Ans. Cube root of $12167 = 23$, and $23 \times 23 \times 23 = 12167$.

The cube root sign is the same as the other, except that a small 3 is placed in it as, $\sqrt[3]{1728} = 12$, meaning that the cube root of $1728 = 12$. In a similar way the cube of a number is written $12^3 = 1728$.

In pointing off decimals, the first point is over the third figure to the right of the decimal point, and in the answer there will be as many decimal places as there are periods pointed off of the decimals.

EXAMPLES

What is the cube root of 9261? *Ans.* 21.

What is the cube root of 1030301? *Ans.* 101.

What is the cube root of 313? *Ans.* 6.789+.

What size must a cubical tank be to contain 226981 cubic inches? *Ans.* 61 inches each way.

How much less is the cube root of 1328 than the square root of the same number? *Ans.* Cube root is 10.9917293. Square root is 36.4417343, cube root is 25.450005, less than the square root.

CHAPTER XIV

MEASURING SURFACES

THE measurement of surfaces of any kind, or the calculation of the contents of any box or tank, or the weight of any bar or other material, brings us to what is called mensuration.

If we have a square piece of tin one inch each way, as in Fig. 1, it is clear that it must measure one square inch in area, as the surface is called. If we cut a square piece measuring two inches each way, as in Fig. 2, and divide it off into one-inch squares, we see at once that it is the same as four squares of one inch each and contains four square inches. This brings us to the first rule, that the area of any rectangle, which means any figure having four square corners, as a square or the oblong Fig. 4, can be found by multiplying the length of one side by the length of one end. In the case of a square this means multiplying one side by the other, and in Fig. 2 we have $2 \times 2 = 4$ square inches.

One thing that is a little puzzling at first is finding the area of a square that is less than one. Suppose the square is only $\frac{1}{2}$ an inch on each side. Multiply $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, and we see that a square of this size only has $\frac{1}{4}$ of a square inch area, as can be seen in Fig. 1, where four $\frac{1}{2}$-inch squares are laid off in the one-inch square. If the side of the

square is 3 inches, the area will be $3 \times 3 = 9$ square inches, and $4 \times 4 = 16$ inches for a four-inch square. This also shows us what is known as the "law of squares," which means that areas of squares do not vary directly with the diameters or sides, and that a 4-inch square is not twice as large as a two-inch square, but 4 times as large. Let us see. The 2-inch square is $2 \times 2 = 4$ square inches in area, and the 4-inch square is $4 \times 4 = 16$ square inches, and 16 is 4 times 4.

Multiplying a number by itself is called "squaring" it, so we see that 4 is the square of 2 and 16 is the square of 4. We also see that the side of the large square is twice as long as the small one, and to compare the areas we divide the large dimension by the small one and square the answer. This gives 4 divided by $2 = 2$, and $2 \times 2 = 4$, showing that the 4-inch square is 4 times as large as the 2-inch. To compare a 2- and a 10-inch square divide 10 by 2, which gives 5, and $5 \times 5 = 25$ times as large. To prove this, multiply $2 \times 2 = 4$ and $10 \times 10 = 100$ and it is easy to see that the 10-inch square is 25 times as large as the 2-inch square. This law holds good for any regular shaped figures if both are exactly alike, except for size, and only similar dimensions are considered. This will be taken up later.

Fig. 3 shows an L-shaped figure which can be calculated best by dividing into two parts, as shown by the dotted line. This makes it a 1-inch square and a rectangle 1×3 inches and $1 \times 3 = 3$, plus $1 = 4$ the total area. Fig. 4 contains the same area in a straight strip 1 inch wide by 4 inches long.

Coming back to the law of squares, we find this very useful in many ways. It shows us that a sheet of iron 2

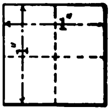


FIG. 1

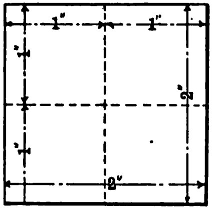


FIG. 2

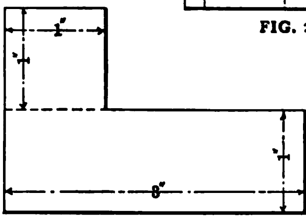


FIG. 3

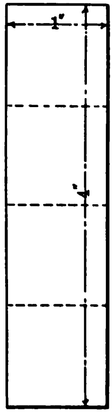


FIG. 4

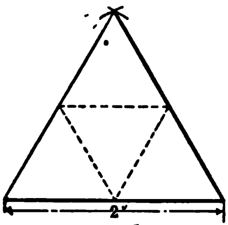


FIG. 5

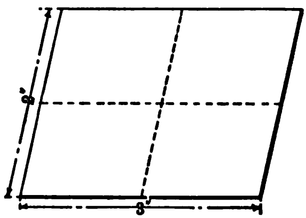


FIG. 6

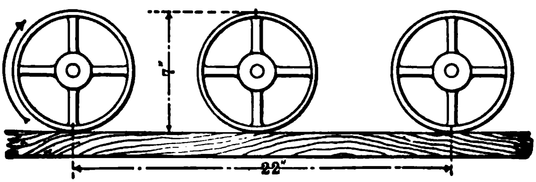


FIG. 7

Surfaces to be Measured.

inches square is 4 times as heavy as a piece 1 inch square of the same thickness, and that a bar 8 inches square is 16 times as heavy as one 2 inches square of the same length. This also applies to circles, to round bars, to triangles, or any other shaped figure. If the side of a triangle is 2 inches and the same side of another one exactly like it is 1 inch, we know that the small one is only $\frac{1}{4}$ as large as the other, as shown in Fig. 5, or in the case of the rhombus or slanting figure in Fig. 6, where the squares of similar dimensions show the comparative areas.

The Circle

The circumference and area of a circle are always a fixed relation to the diameter, and knowing this we can always find the desired quantity very easily. The circumference, or distance around, is practically $3\frac{1}{7}$ times the diameter, 3.14159 to be exact, but 3.1416 is generally used.

To see just why this is so, take a 7-inch pulley and make a mark on the side of the rim and a mark on the bench. Put these marks together as at the left of Fig. 7, and roll the pulley to the right until the mark comes down to the bench again, which measures the distance around the pulley. Mark this point on the bench and measure the distance between the marks. If the pulley is just 7 inches and the work has been carefully done, the marks will be 22 inches apart, which is just $3\frac{1}{7}$ times the diameter of the pulley. If the pulley was 14 inches the distance will be 44 inches, and so on. Figuring the usual way, we have $7 \times 3.1416 = 21.9912$, which is only 88 ten thousandths of an inch different from the other way.

The area of a circle can be figured in three different ways,

whichever happens to be the most convenient. The usual way is to square the diameter and multiply by .7854, because it has been found that a circle is just .7854 as large as a square of the same diameter, the corners being .2146 of the whole area. This means that if a square was 10 inches each way or had an area of 100 square inches, the largest circle that could be cut from it would be 78.54 square inches. If you have any doubts, just make a square

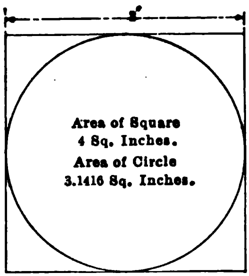


FIG. 8

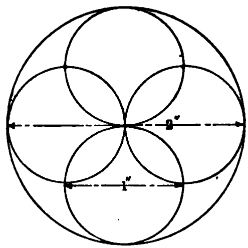


FIG. 9

and a circle of the same diameter and thickness, and get your drug-store friend to weigh them both on his finest scales. You will find them in this proportion if you figure out the weight of each.

The other ways of figuring are:

"Multiply half the circumference by half the diameter."

This may be handy if you know the circumference, as it does away with squaring the diameter, and

Square the radius and multiply by 3.1416.

This is not as apt to be used as the other two.

Fig. 8 shows a 2-inch square with a circle inside of it, and in this case the area is the same as the multiplier for

finding the circumference. A word of warning here may not be amiss, for it is very easy to "discover" relationships which only exist for diameters of 2 and 4, such as the area of a 4-inch circle is 12.56, which is the same as the circumference of a 4-inch circle, except that one is square and the other linear inches or inches of length. But this does not hold true in any other case, and is simply a case of "happen so."

The law of squares applies just the same to circles as to squares, as can be figured out, and can be seen in Fig. 9. Here the small diameter is 1 inch and the large one 2 inches or twice as large. Then as the diameter of the large is twice or 2 times as great, the area will be $2 \times 2 = 4$ times as much, just as in the case of squares. The places where the circles lap just make up for the four open spaces between the small circles and the big one outside.

CHAPTER XV

CONTENTS OR VOLUME OF SOLID BODIES

THE area of any flat surface depends on two dimensions, hight and breadth, and the answer is in square measure, inches, feet, or other units. The volume or cubical contents brings in the third dimension of thickness, and we first find the area in square inches by multiplying the hight by the breadth, and then multiply this by the thickness, in the case of a disk, or the length, in the case of a bar or long piece.

In Fig. 1 we have a plate 5 inches square and 1 inch thick, and, as can be seen by the dotted lines, the front surface contains $5 \times 5 = 25$ square inches. As it is 1 inch thick we multiply this by 1 and get 25 cubic inches, showing that it has the same volume as 25 cubes each 1 inch on all sides. In Fig. 2 we have added four more plates like Fig. 10, and now have a block that is 5 inches on all sides, making it a perfect cube. It is very plain that this cube contains 5 times as many 1-inch cubes as the plate in Fig. 10, and shows that the cubical contents is the hight $\times$ breadth $\times$ thickness or length, as $5 \times 5 \times 5 = 125$ cubic inches.

This also shows that the "law of squares" does not hold, but gives place to the "law of cubes" when the dimensions increase in all three dimensions. In Fig. 1, the side

is 5 times the length of one of the cubes, and as $5 \times 5 = 25$, the area is 25 times as great. But in Fig. 2 we see that the small square which is heavily outlined is only $\frac{1}{125}$ of the whole cube, because $5 \times 5 \times 5 = 125$, the same as the volume.

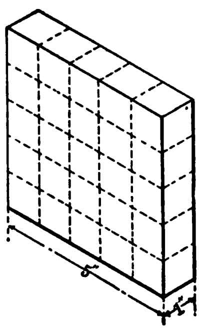


FIG. 1

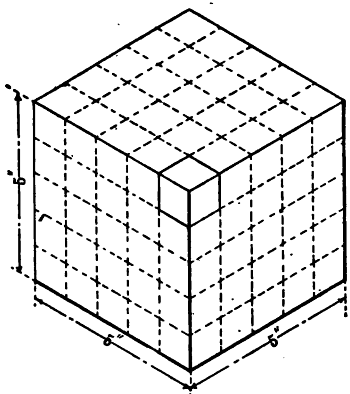


FIG. 2

This is also true of a ball or sphere where all dimensions increase in the same ratio, and a 5-inch ball will weigh 125 times as much as a 1-inch ball.

When, however, one of the dimensions remains the same the law of squares still holds, and this is used more than the other.

Take, for example, two shafts of the same length, but one 2 inches in diameter and the other 4. As 2 into $4 = 2$ and $2 \times 2 = 4$, the large one weighs 4 times as much, though only twice as large in diameter. Or if one shaft is 2 inches and the other 3 inches, how much heavier is the large one, both being the same length? Here we can say

$2 \times 2 = 4$ and $3 \times 3 = 9$. Dividing 9 by $4 = 2\frac{1}{4}$, so that the 3-inch shaft weighs $2\frac{1}{4}$ as much as the small one, which is $\frac{3}{2}$ as large. Or we could say 2 into $3 = 1\frac{1}{2}$, and $1\frac{1}{2} \times 1\frac{1}{2} = 2\frac{1}{4}$, which is the same as before. If the lengths vary the law of cubes can be used, but it is generally easier to find the volume or weight for one unit (inch or foot) of length and multiply each shaft by its length. This avoids confusion and is easily done.

Finding Weights of Bodies

The weight of any body can be figured if we can find the number of cubic inches it contains, and know the weight of one cubic inch of the material. The latter is easier than the first, as the figures given in the table that follows are from good authorities and represent average conditions.

A plate of steel 12 inches square and $\frac{1}{8}$ inch thick contains $12 \times 12 \times \frac{1}{8} = 18$ cubic inches. Multiplying this by .28, the weight of one cubic inch of average steel, we have 5.04 pounds. This shows that for rapid estimating we can remember that a square foot of $\frac{1}{8}$-inch steel weighs 5 pounds, so that we know a plate $\frac{1}{16}$ inch thick will weigh $2\frac{1}{2}$ pounds and one $\frac{5}{8}$ inch thick will weigh $5 \times 5 = 25$ pounds.

If the steel is a bar $\frac{1}{2} \times 3$ inches and 30 inches long, we have $\frac{1}{2} \times 3 \times 30 = 45$ cubic inches $\times .28 = 12.60$ pounds, and any regular bar can be estimated in the same way. A round bar 2 inches in diameter by 3 feet is handled in this way: First find the area of the end, and $2 \times 2 \times .7854 = 3.1416$ square inches. Multiply by 36 and get $3.1416 \times 36 = 113.0976$ cubic inches. This multiplied by .28 gives 31.667 pounds as the weight.

For hollow bodies such as cylinders, it is easiest to subtract the area of the inside hole from the area of the outside, and multiply this difference, which is the area of the ring, by the length. Remember that it makes no difference whether the hole is central or not, as in an eccentric the area is the same. Always be sure that all the dimensions are in the same denomination, as we sometimes get into trouble by multiplying inches by feet or pounds by tons. They must all be alike, either by changing the larger to the smaller or the reverse.

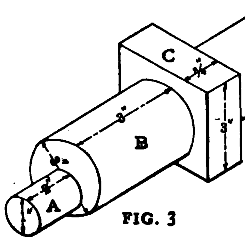


FIG. 3

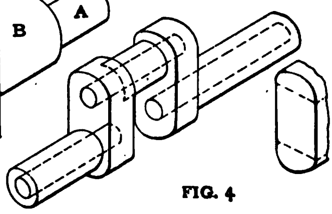


FIG. 4

Where bodies are regular there is very little trouble in finding their contents and weight, and when they are not uniform some little ingenuity is necessary to find just how much they contain. This can usually be done by dividing the body into two or more parts and taking the areas of each part separately. Figs. 3 and 4 show by dotted lines how this can easily be done.

Figs. 3 and 4 show two cases which are about as complicated as we are likely to find. The first is a 2-inch shaft with a square in the center 3 inches on a side by $\frac{3}{4}$ inch wide or thick, each 2 inch diameter is 3 inches long, and on each end is a piece 1 inch in diameter by 2 inches long.

These ends, *A*, are eccentric to the shaft *B*, but that makes no difference in the volume or the weight.

First calculate the volume or contents of the square *C*, $3 \times 3 \times .75 = 6.75$ cubic inches. The area of a 1-inch piece is .7854, and this is 2 inches long, so each end is $2 \times .7854 = 1.5708$ cubic inches, and the two ends are twice this or 3.1416 cubic inches. The 2-inch bar has an area four times the 1-inch or 3.1416, and multiplying by its length we have $3 \times 3.1416 = 9.4248$ cubic inches in each piece. Both of these will contain $2 \times 9.4248 = 18.8496$ cubic inches, and we add $6.75 + 3.1416 + 18.8496$ and get 28.7412 cubic inches in the shaft. To find the weight we multiply by the weight of one cubic inch of the material it is made of, as found in the table on following page.

Fig. 4 is a crank-shaft, and is given to show how it can be divided up into parts in the same way. The crank-pin between the cheeks are simply round bars with holes bored in them, and can be easily figured. The cranks have round ends and can be divided as shown in the sketch at the side. This makes it two half circles and a square block, so that the two ends can be figured together as a circle and added to the contents of the block in the center.

A GENERAL RULE FOR ALL REGULAR SOLIDS

In many works on Mensuration, designed for schools, and in a few of the "Table and Data" books compiled for the use of engineers, there is given a rule for ascertaining the volume of certain solids called "Prismoids" which are not usually named in the category of regular bodies, but for which, as for them, a ready method of solution is equally necessary. These prismoidal forms have flat ends, which

PROPERTIES OF METALS

Metal	Melting Point	Wt. per Cu. In.	Wt. per Cu. Ft.	Tensile Strength	Specific Gravity	Chemical Symbol
Aluminum	1157	.0924	159.63	20,000	2.56	Al.
Antimony	1130	.2424	418.86		6.71	Sb.
Bismuth	505	.354	611.76		9.83	Bi.
Brass, cast	1692	.3029	523.2	24,000	8.393	
Bronze	1692	.319	550.	36,000	8.83	
Chromium	3500	.2457	429.49		6.8	Cr.
Cobalt	2732	.307	530.6		8.5	Co.
Copper	1929	.322	556.	36,000	8.9	Cu.
Gold	1965	.6979	1206.05	20,000	19.32	Au.
Iridium	3992	.8099	1400.		22.42	Ir.
Iron, cast	2700	.26	450.	16,500	7.21	Fe.
Iron, wrought	2920	.278	480.13	50,000	7.7	Fe.
Lead	618	.41	710.	3,000	11.37	Pb.
Manganese	3452	.289	499.4		8.	Mn.
Mercury	— 39	.4909	848.35		13.59	Hg.
Nickel	2700	.3179	549.34		8.8	Ni.
Platinum	3227	.7769	1342.13		21.5	Pt.
Silver	1733	.3805	657.33	40,000	10.53	Ag.
Steel — cast	2450	.28	481.2	50,000	7.81	
Steel — rolled	2600	.2833	489.6	65,000	7.854	
Tin	445	.2634	455.08	4,600	7.29	Sn.
Tungsten	3600	.69	1192.31		19.10	W.
Vanadium	3230	.1987	343.34		5.50	V.
Zinc	779	.245	430.	7,500	6.86	Zn.

are parallel to each other, and they have three or more sides, each one of which is a geometric plane, but the ends may be very dissimilar in shape and very different in area; for this reason a special rule has been devised and demonstrated, and is entered on the list of rules for solids under the name of the "Prismoidal Formula."

The exact form and working of this rule may be given in the following words; all dimensions must be taken in units of a like kind:

Add together $\left\{\begin{array}{l}\textit{The area of the base.}\\ \textit{The area of the top.}\\ \textit{Four times the area of the middle section.}\end{array}\right.$

Multiply this sum by one sixth of the perpendicular hight. The resulting product is the cubic contents, or volume required.

The *middle section* must of course be taken midway between the ends and parallel thereto, and its elements found either by calculation or by some graphic method.

The prismoidal rule is serviceable in calculating the cubic contents of many regularly formed bodies. For the rectangular solids, and for cylinders, cones and pyramids, the usual rules given are shorter in the calculation than this one, but then it is well to know that the prismoidal formula will meet and serve all cases.

For the sphere, the difference of figuring may be little; as for instance, take one twelve inches in diameter to find its volume. The usual rule is: *Multiply the cube of the diameter by* .5236; thus: $12 \times 12 \times 12 \times .5236 = 904.78$.

The prismoidal rule works the problem out in this way:

Area of the top	= 000.
Area of the bottom	= 000.
Four times the middle area	= 452.39
Multiply by one-sixth the hight ...	2
	904.78

This rule, as applied to the sphere, resolves itself into the product of one sixth its diameter, and the area of a circle whose diameter is double the given diameter of the sphere.

Four times the middle section here is simply the area of a

circle twenty-four inches diameter, a method of procedure in accordance with the mathematical law, which proves that a double diameter gives a quadruple area. With the help of the area tables, in the latter method, the arithmetical process of finding the volume of the sphere is really shorter than the usual way of cubing its diameter, and then multiplying the product so found by the "regulation" decimal.

Many individual rules which facilitate solution are based on certain leading dimensions given in the following:

$$\left.\begin{array}{c}\text{The volume of}\\ \text{a sphere}\end{array}\right\} = \begin{cases}\text{Surface} \times \frac{1}{6}\text{ diameter.}\\ \text{Radius}^3 \times 4.1888.\\ \text{Diameter}^3 \times .5236.\\ \text{Circumference}^3 \times .0169.\end{cases}$$

These are examples of *one rule for each*, but how much more convenient to the memory is the application of the *one rule for all*, to this group of bodies, as it is to many other forms.

To express a general rule for certain bodies in terms differing from any of these, the following is offered: *The volume of any figure generated by a revolving surface is equal to the product of the area of the generating surface and the circumference described by its center of gravity.*

Let us give an illustration and proof of this last method. A cylinder, eight inches in diameter, and eight inches long, is generated by the revolution of a rectangle four inches by eight inches about its longer side, as axis; that is, the space swept through by this revolving rectangle is that of a cylinder, the dimensions of which are given. The center of gravity of this rectangle is the center of figure, and the circumference described by this point, in revolution, is 12.5664

inches corresponding to a diameter of four inches. We have, therefore, $4\times8\times12.5664=402.1248=$ the volume required.

By the usual rule for cylinders we have the area of the base multiplied by the length of the axis, or the hight, thus:

Area of eight-inch diameter = 50.2656, which, multiplied by 8 = 402.1248, the same as the other.

By the prismoidal formula we have:

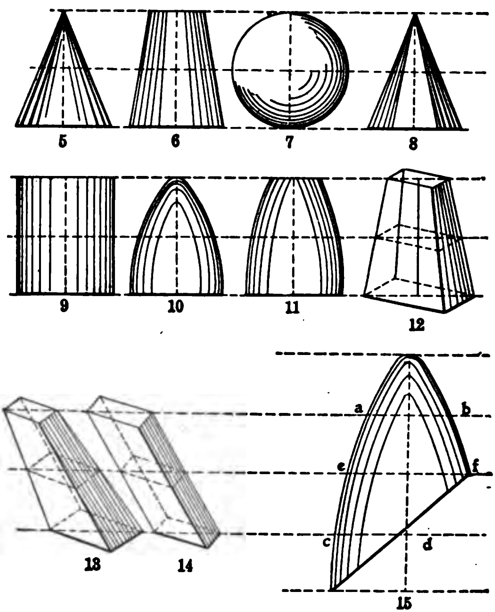
Area of top	50.2656
Area of bottom	50.2656
Four times area of middle	201.0624
The sum of these	301.5936
Multiplied by $\frac{1}{6}$ of hight, or, which is the same, add	100.5312
	402.1248

From an inspection of this latter array of figures, it plainly appears that the result sought, and here found, is eight times the area of one end, but which shows something in favor of the prismoidal formula.

There are many forms of revolution resembling a shallow dish or deep kettle, resulting from a sweeping fragment of a curve, such as soap pans, stills, retorts, and the like, the contents of which are always wanted to be known, and the weight of the vessels also, before they are cast.

The inner capacity found in cubic inches may be converted into gallons, and the difference between the inner and outer "skins" of the vessel, representing the thickness (which may not be uniform), may be counted in cubic inches, from which the weight can be figured. If the curve varies, and close results are needed, the body may be divided into zones, and the rule applied as directed, to each segment.

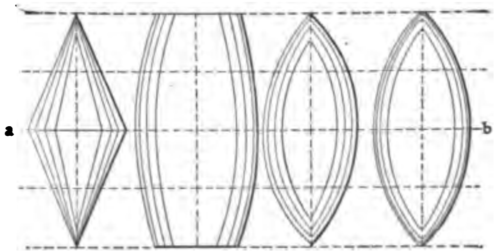
To make the application of the "one rule for all" more clear to the reader, we give some illustrations of regularly formed solids.



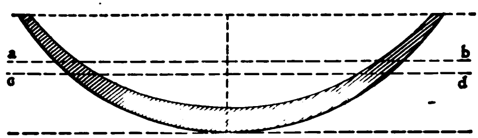
From the examples already given in the text, and referring in this place to Figs. 5 to 14 as above, no modification of the rule in its use for ascertaining their solid content is necessary, the lines passing the areas of top and bottom and middle are plainly indicated.

The top areas of Figs. 5, 8 and 10, and the top and bottom areas of Fig. 7, being equal to 0. In every case the perpendicular, not the slant, hight must be taken.

In Fig. 15, the part above *e f* must be treated the same as Fig. 10, in which *a b* is the middle section; and the part below *e f*, in which *c d* is the middle section, must be calculated separately, the bottom area being equal to 0. The sum of the solidities of the two parts being, of course, equal to the whole.



16 17 18 19



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Figs. 16, 17, 18 and 19 must be halved by the medial line *a b*, and each half calculated in the same way as directed for Figs. 5, 10 and 11, which they resemble in form.

In Fig. 20 we have the section of a dish or pan. The line *a b* cuts the middle section of the inner skin or content of the pan, so to speak, and the line *c d* the middle section of the outer skin. The space between the two skins represents the cubic content of the material of the pan, and the cubic content of the inner skin will represent the capacity of the pan; this latter may be converted from units of lineal

measure into gallons, which answers the question as to what it will hold, and the former into pounds weight, from which the price can be found, thus readily solving the dual problem. Other regularly formed solids will yield to the same treatment.

CHAPTER XVI

MEASURING ANGLES

ANGLES are sometimes puzzling and not thoroughly understood. An angle is the measure of the inclination of one line to another, and is based on a circle divided into 360 equal parts. This is purely arbitrary, but makes a very good system when thoroughly understood. The first thing is to realize that a degree is not a measure of distance, but is $\frac{1}{360}$ of the circle that we are working with and is really a measure of the inclination of lines as stated before. Another thing is, that if you are measuring degrees of any circle by length, the measure must be taken on the circle and not across the corners, and it will be clear that the length of the degree will vary with the diameter of the circle as it is not a measurement of length but of $\frac{1}{360}$ part of a circle.

Fig. 1 shows three circles around a given center, each circle being divided into four parts by the solid lines and into eight parts by the dotted lines. The four solid lines divide it into four equal parts so that each quarter contains one fourth of 360 degrees, or 90 degrees. Now the distances around the circles vary with each diameter, but the angle is the same in each, as can be clearly seen. Dividing each of these 90-degree angles in two equal parts gives 45 degrees for each, and as each part is one eighth of the

whole circle, we can divide 360 by 8 and get 45 degrees just the same.

Similar angles always bear the same proportion to each other without regard to the size of the figures, as can be seen in Fig. 1, and lines at right angles to each other always produce 90 degrees of angle whether they are one inch or one mile in length. In Fig. 2 we have three triangles of different sizes, one within the other, yet it can be seen that the angles are the same in each case, and where the angles

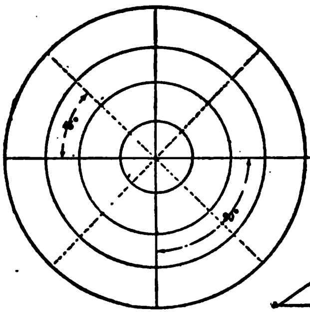


FIG. 1

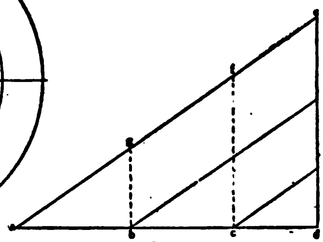


FIG. 2

are the same, the lengths of all sides vary just in proportion to the length of any one side. Thus, if *a d* is 3 inches and *a c* 2 inches, then all the sides of the triangle *a c f* will be two thirds as long as the sides of *a d e*. And if all the sides of any triangle are exactly in proportion the angles must be the same in both.

Going a step farther, we find that the sum of the angles of any triangle is always 180 degrees, the same as in a half circle. Thus an equilateral triangle with all angles equal must be 60 degrees in every angle. If you have a triangle

having two equal sides and know the central angle, the other two can be easily found. With two sides equal, the two outer angles must be the same, so we subtract the central angle from 180, and half the remainder must be the angle of the other two corners. With two angles of a triangle given, the third angle can be found by adding the two together and subtracting this from 180 degrees.

Triangles

Four forms of triangles will show how this works. Fig. 3 is an equilateral or equal triangle, each angle being 60 degrees or 180 in all. Fig. 4 is a right-angled triangle with the other angles 35 and 55 degrees. Fig. 5, a triangle with

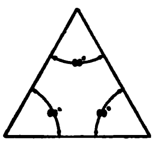


FIG. 3

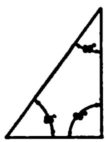


FIG. 4

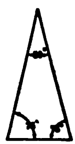


FIG. 5

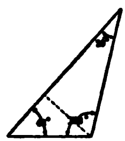


FIG. 6

two equal sides and angles of 75 degrees, the remaining angle being 30 degrees. Fig. 6 is unequal in every way, one being 29 degrees, another 48 degrees, and the large angle 103 degrees. This also shows how any triangle can be divided into right-angled triangles to find the area or for other purposes requiring a right triangle. A line drawn at right angles to the long side to the opposite point is the easiest way with a triangle having all sides unequal. With two sides equal, as Fig. 5, the line can be drawn at right angles to the short side and to the opposite point, in this case the top.

Setting Compound Rests

The difficulty experienced in measuring angles and setting dividing heads and compound rests comes mainly from two causes: A confusion of ideas as to whether half or the total angle is meant and the position of the base line.

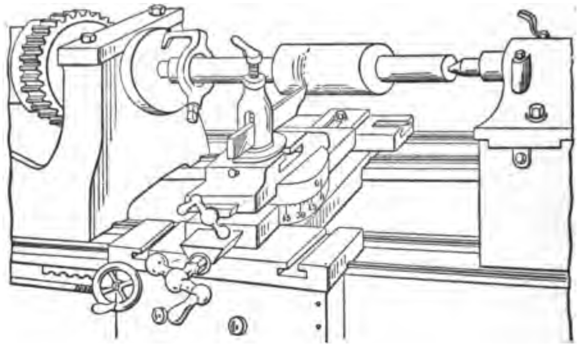


FIG. 7. — Compound Rest on Lathe.

In the compound rest we have the work measured in angles from a line drawn between the lathe centers, while the slide is at right angles to this as in Fig. 7. If we have to turn the bevel on a valve-seat reamer that is 60 degrees total angle, as is usual, we must set the slide rest at 30 degrees from the line of the centers. This very seldom means that we can set it to 30 degrees on the compound rest as they are usually divided to read from the cross-movement at right angles to the center.

Perhaps the easiest way is to swing the compound rest parallel to the lathe centers with the handle toward the headstock, and the two 45-degree marks come together if

they are divided in this way. Then move the handle end out until 30 degrees have passed by the 45-degree mark, or until 15 degrees on the upper coincides with the 45 if both are graduated. No matter how it is divided, move the compound 30 degrees without regard to the numbers as they appear.

When you are facing work to any desired angle and the work is normally in line with the cross-slide, you can read the divisions just as they are graduated, bearing in mind that each degree the slide is set off means 2 degrees total angle for the work.

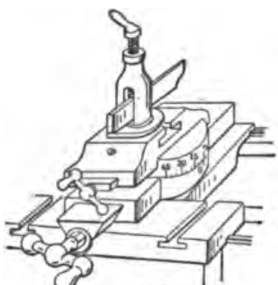


FIG. 8

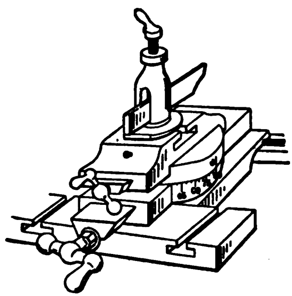


FIG. 9

Different Graduation.

Figs. 7, 8, and 9 show three methods of graduating compound rests on a lathe or swivel head on a planer. In Fig. 7 the base is divided into 45 degrees each side of a zero line at the side or at right angles to the cross-slide. With this graduation the scale shows the degrees moved through by the tool slide with reference to the cross-slide. If we set it to 15 degrees we can face off a piece 15 degrees on each side of the end, but this would leave the end with

a total angle of 150 degrees with the center line of the work, as seen in Fig. 10.

In Fig. 8 the graduations are reversed, being on the upper slide, and the zero on the base. The results are the same, except that we read on the opposite side of the zero mark of the graduations; swinging the upper slide to the left 15 degrees, we must read the angle on the side of the scale now hidden from view.

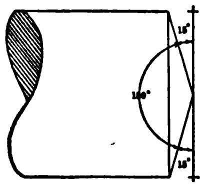


FIG. 10

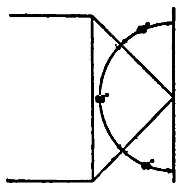


FIG. 11

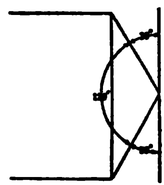


FIG. 12

Fig. 9 is a different plan and one which has some things to recommend it. The 90-degree mark is in place of the zero, in Fig. 7, the 75-degree in place of the 15, but, of course, the 45 comes in the same place, an account of its being half-way between the two.

This method of graduation shows the exact angle that will be cut each side of the center line, and we get the total angle by doubling the figures of the graduation. If we move it to 75 it will cut the same as the other rest did in Fig. 7, and the result will be 75 degrees each side of the center line, as in Fig. 10. If it is moved to 45 it cuts a 90-degree total angle, as in Fig. 11, and if to 60 it cuts 30 degrees away on each side, leaving 120 degrees included angle, as in Fig. 12. Bearing this in mind, there should be no confusion as to what angle will be cut.

After one becomes familiar with the divisions of the machine he is handling, it is easy to set by the numbers, but as they are apt to be confusing, it is safer to check the reading by counting the degrees from the point when the side is parallel with the lathe centers.

CHAPTER XVII

MAKING AND USING FORMULAS

A FORMULA is simply a rule in which the various quantities are represented by letters, and the operations to be performed shown by signs and the position of the letters. There is nothing to be afraid of about a formula, and they can be handled as easily as a regular rule if we understand a little about them. The best way to understand a thing is to make it and see how it goes together, then we can know if anything goes wrong.

Taking an easy rule, as the one for pulley speeds, we know: Speed of Driven Pulley = Multiply the diameter of the driving pulley by its speed and divide this by the diameter of the driven pulley.

To turn this into a formula we let

D represent Diameter of Driving Pulley.
d represent Diameter of Driven Pulley.
S represent Speed of Driving Pulley.
s represent Speed of Driven Pulley.

Then we say:

$$s = \frac{D \times S}{d}$$

showing that D times S are divided by d instead of the whole wording of the rule.

Calling the Driving pulley 20 inches in diameter, the

Driven pulley 10 inches, and the speed of the Driver 200 revolutions a minute, we put these values in place of the letters and have $s = \frac{20 \times 200}{10} = \frac{4000}{10} = 400$ revolutions per minute for the speed of the driven pulley.

When two or more letters, each representing a different quantity, are placed together without any sign between them, it is understood that they are to be multiplied together the same as though the sign of multiplication was between them. All other signs are always shown.

So we often see the formula for horse-power written

$$\text{H.P.} = \frac{\text{PLAN}}{33000}$$

meaning that P or mean effective pressure, L or length of stroke in feet, A or area of piston in square inches, and N or the number of strokes per minute, are all multiplied together and divided by the number 33000 to find the horse-power. This 33000 is called a constant, the same as 3.1416 is the constant representing the ratio between the diameter and circumference of a circle, and .7854 is the constant ratio between the area of a circle and a square having its side the same diameter as the circle.

Coming to a little more complicated formula, we have the one for translating Fahrenheit to Centigrade thermometer. In Centigrade 0 is freezing and 100 is boiling, while in Fahrenheit 32 is freezing and 212 is boiling, so that 212 − 32, or 180 degrees of the Fahrenheit scale, equals 100 degrees of the Cent. scale. We have Cent. = Fahr. − $32 \times \frac{5}{9}$ or Cent. $= 212 - 32 \times \frac{5}{9} = 212 - 32 = 180 \times \frac{5}{9}$

$= 100$, and Fahr. $=$ Cent. $\times \frac{9}{5} + 32$, or $100 \times \frac{9}{5} = \frac{900}{5} = 180 + 32 = 212$.

Wherever plus or minus signs come in, be sure to perform these operations in their proper turn or there will be trouble. If in the first case we took $\frac{5}{9}$ of 212 and then subtracted 32, the answer would be 85.7 instead of 100. In some cases the formula is written with the numbers that belong together enclosed in brackets and that is a very good way, as,

$$P = \frac{7200}{D}\left(T - .333\left(1 - \frac{D}{100}\right)\right) - 100$$

the formula for cast-iron pipes from Kent's Pocket-Book, where P = pressure, D = inside diameter, T = thickness of the shell. Calling D = 10 and T = 2, and beginning with the inside brackets, we have $1 - \frac{10}{100} = 1 - \frac{1}{10} = .9$. Next we have $2 - .333 = 1.677 \times .9 = 1.5093$ for the value of the number inside the brackets. This, then, is to be multiplied by $\frac{7200}{10} = 720 \times 1.5093 = 1086.696$, and from this we subtract 100 pounds, leaving 986.696 as the safe working pressure for a cast-iron pipe 10 inches inside diameter and 2 inches thick. So while the formula looks rather complicated, it is easily handled if we take one step at a time.

Transposing Formula

One of the handy features of using formulas is the ease with which they can be transposed to find any particular factor you desire, much more easily than with a rule in regular form.

Taking the simple case of the formula for electric current,

$C = \frac{E}{R}$, or current equals volts divided by resistance, we can see how to handle more complicated formula. Perhaps the easiest way to thoroughly understand just how to handle this is to consider the whole thing as an example in division, so that the result desired is the quotient, the numerator of the fraction is the dividend, and the denominator is the divisor. We know that the dividend must equal the divisor and quotient multiplied together, and as dividing the dividend by the divisor gives the quotient, so by reversing this and dividing the dividend by the quotient we must get the divisor.

Trying this out with the formula we have, we start with $C = \frac{E}{R}$, in which E is the dividend, R the divisor, and C the quotient. Then R, the divisor, must equal $\frac{E}{C}$, and E, the dividend, equals both R and C multiplied together, or $R = \frac{E}{C}$ and $E = R \times C$.

Giving these values, $E = 100$ volts, $R = 50$ ohms resistance, what is C? Then $C = \frac{100}{50} = 2$. To find R we say, $R = \frac{E}{C} = \frac{100}{2} = 50$, and to find E we multiply $R \times C$, and have $E = 50 \times 2 = 100$, proving that this is right. You can always prove these things, to make sure they are right, and unless you are perfectly sure, it is best to do so.

When the formula becomes more complicated this transposing is not quite so easy, but the same principles hold true and will work out in every case.

The formula for falling bodies is, $h = \frac{g t^2}{2}$, where

h = hight in feet from which a body falls;
g = acceleration per second = 32.16 feet;
t = number of seconds in falling,

will show us how this works out. As g is given, the only thing to find is t^2 or t, which is one factor of the dividend. To make this clear, suppose we had to divide 100 by 10. The answer is clearly 10. Now call the dividend 25×4 instead of 100, and suppose we wish to find one of these factors, the other being known. Multiplying divisor and quotient gives the whole dividend, so dividing the product of the divisor and dividend by the known factor of the dividend will give the other factor. This means that

$$t^2 = \frac{h \times 2}{g}, \text{ or } t = \sqrt{\frac{h \times 2}{g}}.$$

Trying this transposition, we have first:

$$h = \frac{32.16 \times 100}{2}, \text{ where } t = 10 \text{ seconds and } t^2 = 100.$$

$$h = \frac{3216}{2} = 1608 \text{ feet in 10 seconds.}$$

Transposing we have:

$$t^2 = \frac{h \times 2}{g} = \frac{1608 \times 2}{32.16} = \frac{3216}{32.16} = 100.$$

$$t^2 = 100,\ t = 10.$$

Going a step farther, we take the wiring formula, $R = \frac{D \times 1000}{C \times 2 L}$ where R = resistance in ohms per 100 feet, D = volts drop, C = amperes, L = single length of line.

To find D we have $D = \frac{R \times C \times 2\,L}{1000}$.

To find C we have $C = \frac{D \times 1000}{R \times 2\,L}$.

To find L we have $2\,L = \frac{D \times 1000}{R \times C}$, and

$$L = \tfrac{1}{2} \text{ of } \frac{D \times 1000}{R \times C}.$$

Calling L = 1000 feet, D = 10 volts, and C = 100 amperes, we have

$$R = \frac{10 \times 1000}{100 \times 2 \times 1000} = \frac{10000}{200000} = \frac{1}{20} \text{ ohm, and}$$

$$D = \frac{\frac{1}{20} \times 100 \times 2 \times 1000}{1000} = \frac{10000}{1000} = 10.$$

$$C = \frac{10 \times 1000}{\frac{1}{20} \times 2 \times 1000} = \frac{10000}{100} = 100.$$

$$2\,L = \frac{10 \times 1000}{\frac{1}{20} \times 100} = \frac{10000}{5} = 2000,$$

$$L = \tfrac{1}{2} \text{ of } 2000 = 1000.$$

Taking another formula, the one for horse-power,

$$H.P. = \frac{P \times L \times A \times N}{33000}, \text{ where}$$

P = effective pressure per square inch.

L = length of stroke in feet.

A = area of piston in square inches.

N = number of strokes per minute or twice the revolutions.

Remembering that the numerator is the dividend divided into four factors, we can find any one of them by taking it

out of the numerator, making the rest into the divisor and putting H.P. × 33000 for the numerator.

This is easily done as follows:

$$P = \frac{\text{H.P.} \times 33000}{L \times A \times N}. \qquad A = \frac{\text{H.P.} \times 33000}{P \times L \times N}.$$

$$L = \frac{\text{H.P.} \times 33000}{P \times A \times N}. \qquad N = \frac{\text{H.P.} \times 33000}{P \times L \times A}.$$

CHAPTER XVIII

THE VERNIER AND MICROMETER

This method of measuring or of dividing known distances into very small parts is credited to the invention of Pierre Vernier in 1631. The principle is shown in Figs. 1 to 3, and its application in Figs. 4 and 5. In Figs. 1 and 2 both distances 0–1 are the same, but they are divided

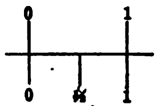


FIG. 1

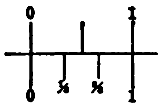


FIG. 2

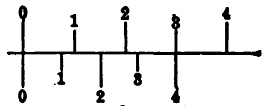


FIG. 3

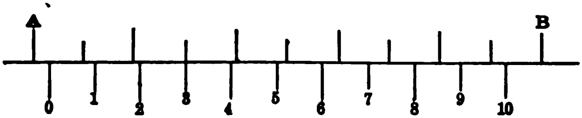


FIG. 4

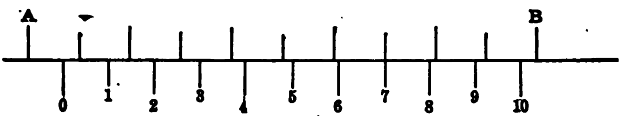


FIG. 5

Vernier Reading.

into different divisions. Calling 0 – 1 = 1 inch, then in Fig. 1 it is clear that moving the lower scale one division will divide the upper one in half. In Fig. 2 the upper scale is divided in half and the lower one in thirds. If the lower

scale is moved either way until $\frac{1}{3}$ or $\frac{2}{3}$ comes under the end line, it has moved $\frac{1}{3}$ of an inch, but if either of these are moved to the center line, then it is only moved $\frac{1}{2}$ of this amount or $\frac{1}{6}$.

Fig. 3 shows the usual application of the principle except that it is divided in four parts instead of ten. Here both the scales have four parts, but on the lower scale the four parts just equal three parts of the upper scale. It is evident that if we move the lower scale so that 0 goes to 1 and 4 goes to 4, that it will be moved $\frac{1}{4}$ the length of the

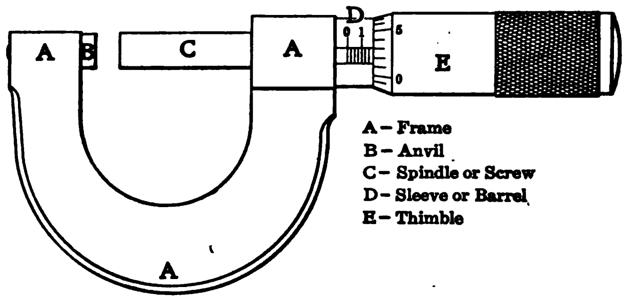


FIG. 6.—Micrometer.

distance 0 — 4 on the upper scale. If this distance was 1 inch, each division on the upper scale equals $\frac{1}{4}$ inch, and moving the lower scale so that the line 1 just matches the line next to 0 on the upper scale gives $\frac{1}{4}$ of one of these divisions or $\frac{1}{16}$ of an inch.

Figs. 4 and 5 show the usual application in which the lower or vernier scale is divided into 10 parts, which equals 9 parts of the upper scale. The same division holds good, however, and when the lower scale is moved so that the

first division of the vernier just matches the first line of the scale, it has been moved just one tenth of a division. In Fig. 4 the third lines match so that it has moved $\frac{3}{10}$, and in Fig. 5 $\frac{7}{10}$ of a division. So if A B is one inch, then each division is $\frac{1}{10}$ of an inch, and each line of the vernier is $\frac{1}{10}$ of that or $\frac{1}{100}$ of an inch.

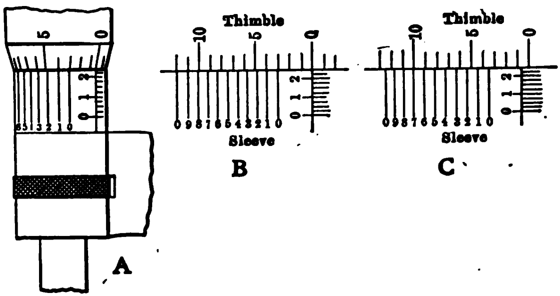


FIG. 7.—Micrometer Graduations.

To find the reading of any vernier, divide one division of the upper or large scale by the number of divisions in the small scale. So if we had a vernier with 16 divisions in each, the large scale being 1 inch long, then the movement of one division is $\frac{1}{16}$ of $\frac{1}{16}$ or $\frac{1}{256}$ of an inch.

Reading the Micrometer

The commercial micrometer consists of a frame, the anvil or fixed measuring point, the spindle which has a thread cut 40 to the inch on the portion inside the sleeve or barrel, and the thimble which goes outside the sleeve and turns the spindle. One turn of the screw moves the spindle $\frac{1}{40}$ or .025 of an inch, and the marks on the sleeve

show the number of turns the screw is moved. Every fourth graduation is marked 1, 2, 3, etc., representing tenths of an inch, or, as each mark is .025, the first four means $.025 \times 4 = .100$, the third means $.025 \times 4 \times 3 = .300$.

The thimble has a beveled edge divided into 25 parts and numbered 0, 5, 10, 15, 20, and to 0 again. Each of these mean $\frac{1}{25}$ of a turn or $\frac{1}{25}$ of $\frac{1}{40} = \frac{1}{1000}$ of an inch. To read, multiply the marks on the barrel by 25 and add the graduations on the edge of the thimble. In the cut there are 7 marks on the sleeve and 3 on the thimble, so we say $7 \times 25 = 175$, plus $3 = 178$ or .178.

In shop practice it is common to read them without any multiplying, by using mental addition. Beginning at the largest number shown on the sleeve and calling it hundreds and adding 25 for each mark, we say in the case shown 100 and 25, 50, 75, and then add the numbers shown on the thimble, 3, making 178 in all. If it showed 4 and one mark, with the thimble showing 8 marks, the reading would be $400 + 25 + 8 = 433$ thousandths or .433.

The Ten-Thousandth Micrometer

This adds a vernier to the micrometer sleeve or barrel, as shown in Fig. 7, which is read the same as any vernier, as has been explained. First note the thousandths as in the ordinary micrometer and then look at the line on the sleeve which just matches a line on the thimble. If the two zero lines match two lines on the thimble, the measurement is in even thousandths as at *B*, which reads .225. At *C* the seventh line matches a line on the thimble, so the reading is .2257 inch.

CHAPTER XIX

REGULAR POLYGONS AND THEIR PROPERTIES

THE easiest way to lay out figures of this kind is to draw a circle and space it off, but it saves lots of time to know what spacing to use or how large a circle to draw to get a figure of the right size. Suppose we wish to lay out any regular figure, such as a pentagon or five-sided figure, having sides $1\frac{1}{2}$ inches long.

Looking in the third column, we find "Diameter of circle that will just enclose it," and opposite pentagon we find

Number of Sides	Name of Figure	Diameter of Circle that will just enclose when side is 1	Diameter of circle that will just go inside when side is 1	Length of side where diameter of enclosure circle equals 1	Length of side where inside circle equals 1	Angle formed by lines drawn from center to corners	Angle formed by outer sides of figures	To find Area of Figure multiply side by itself and by number in this column
3	Triangle ..	1.1546	.5774	.866	1.732	120°	60°	.4330
4	Square	1.4142	1.	.7071	1.	90	90	1.
5	Pentagon ..	1.7012	1.3764	.5878	.7265	72	108	1.7204
6	Hexagon ..	2.	1.732	.5	.5774	60	120	2.5980
7	Heptagon .	2.3048	2.0766	.4338	.4815	51°-26′	$128\frac{4}{7}$	3.6339
8	Octagon...	2.6132	2.4142	.3827	.4142	45	135	4.8284
9	Nonagon ..	2.9238	2.7474	.342	.3639	40	140	6.1818
10	Decagon ..	3.236	3.0776	.309	.3247	36	144	7.6942
11	Undecagon	3.5494	3.4056	.2817	.2936	32°-43	$147\frac{3}{11}$	9.3656
12	Dodecagon	3.8638	3.732	.2588	.2679	30	150	11.1961

1.7012 as the circle that will just enclose a pentagon having a side equal to 1. This may be 1 inch or 1 anything else, so as we are dealing in inches we call it inches. As the side of the pentagon is to be 1½ inches, we multiply 1.7012 by 1½ and get 2.5518 as the diameter of circle to draw, and take half of this or the radius 1.2759 in the compass to draw the circle. Then with 1½ inches in the dividers we space round circle, and if the work has been carefully done it will just divide it into five equal parts. Connect these points by straight lines, and you have a pentagon with sides 1½ inches long.

If the pentagon is to go inside a circle of given diameter, say 2 inches, look under column 5 which gives "Length of side when diameter of enclosing circle equals 1," and find .5878. Multiply by 2 as this is for a 2-inch circle, and the side will be $2 \times .5878 = 1.1756$. Take this distance in the dividers and step around the 2-inch circle.

Assume that it is necessary to have a triangular end on a round shaft, how large must the shaft be to give a triangle 1.5 inches on a side?

Look in the table under column 3, and opposite triangle find 1.1546, meaning that where the side of a triangle is 1, the diameter of a circle that will just enlcose it is 1.1546. As the side is 1.5, we have $1.5 \times 1.1546 = 1.7318$, the diameter of the shaft required. If the corners need not be sharp, probably a shaft 1.625 would be ample.

Reversing this, to find the size of a bearing that can be turned on a triangular bar of this size, look in column 4, which gives the largest circle that will go inside a triangle with a side equal to 1. This gives .5774. Multiply this by $1.5 = .8661$.

A square taper reamer is to be used which must ream 1 inch at the small end and 1.5 at the back, what size must this be across the flats at both places?

Under column 5 find .7071 as the length of the side of a square when the diameter of the enclosing circle is 1, so this will be the side of the small end of the reamer, and $1.5 \times .7071 = 1.0606$ is the side of the reamer at the large end.

The Circle

A circle is a continuous curved line having every point at an equal distance from the center.

Its *perimeter* or circumference is always 3.14159265359 times the diameter, although 3.1416 is generally used and $3\frac{1}{7}$ is a very close approximation.

Area equals the diameter squared $\times$ .7584, or half the diameter squared $\times$ 3.1416, or half the diameter $\times$ half the circumference.

Diameter of a square having equal area = diameter of circle $\times$ .89 very nearly.

The Triangle

Equilateral triangle is a regular figure having three equal sides and three equal angles of 60 degrees each.

The *side* equals .866 times the diameter of enclosing circle.

Distance from one side to opposite point equals the side times .866, or diameter of enclosing circle $\times$ .75, or inside circle $\times$ $1\frac{1}{2}$.

Diameter of enclosing circle equals side times 1.1546, or $1\frac{1}{3}$ times distance from side to point, or twice inside circle.

Diameter of inside circle equals side times .5774 or $\frac{1}{2}$ the enclosing circle.

The area equals one side multiplied by itself and by .433013.

Diameter of circle having equal area equals side of triangle times .73.

The Square

A square is a figure with four equal sides and four equal angles of 90 degrees.

Its *perimeter* or outside surface is four times the length of one side.

Area equals one side multiplied by the other, which is the same as multiplying by itself, or "squaring."

Diagonal or "long diameter," or "distance across corners," equals the side multiplied by 1.414.

Area of circle that will go inside the square equals one side multiplied by itself times .7854, or .7854 times the area of the square.

Area of circle that will just enclose the square equals diagonal multiplied by itself times .7854, or 1.27 times the area of the square.

Diameter of a circle having an equal area is 1.126 or practically $1\frac{1}{8}$ times the side of the square.

The Hexagon

A *hexagon* is a regular figure with six equal sides and six equal angles of 120 degrees. It can be drawn inside a circle by spacing around with the radius of the circle.

The *side* equals half the diameter of enclosing circle.

The *long diameter* equals diameter of enclosing circle or twice the length of one side.

The *short diameter* equals the long diameter multiplied by .866 or 1.732 times one side.

The *area* equals one side multiplied by itself and by 2.5981.

The *area* of enclosing circle is one side multiplied by itself and by 3.1416.

The *area* of an inside circle is the short diameter multiplied by itself and by .7854.

Diameter of circle having equal area is practically .9 times long diameter.

The Octagon

An *octagon* is a regular figure with eight equal sides and eight equal angles of 135 degrees.

The *side* equals the long diameter multiplied by .382.

The *side* equals the short diameter multiplied by .415.

The *long diameter* equals diameter of enclosing circle or one side multiplied by 2.62.

The *short diameter* equals the long diameter multiplied by .93, or one side multiplied by 2.45.

The *area* equals one side multiplied by itself and by 4.8284.

The area of enclosing circle is 1.126 times area of octagon.

The area of inside circle is .972 times area of octagon.

The diameter of a circle having equal area is .953 times the long diameter of the octagon.

What is length of the side of a pentagon drawn in a circle with a diameter of 40 inches? Look in column 4 opposite pentagon or 5, and find .5878. Multiply by 40 and get 23.5114 inches as side of pentagon.

To find its area, square the side and multiply it by the number in column 9 which is 1.7205, and get 929 square inches as the area.

Or having the area of an octagon, to find the side. Divide the area by 4.8284 (col. 9) and take the square root of answer.

Having the side, find the diameter of circle which will just touch its corners by dividing by 0.76536.

Polygons drawn outside of a circle can be handled in the same way by using columns 3 and 5.

CHAPTER XX

THE USES OF SHOP "TRIG"

THE laying out of angles is sometimes difficult by ordinary methods and a little knowledge of shop "trig" is very useful and much easier than as though we called it by its full name.

It is really a system of constants or multipliers based on the fact that there are always fixed proportions between the sides and angles of triangles and other figures. Fig. 1

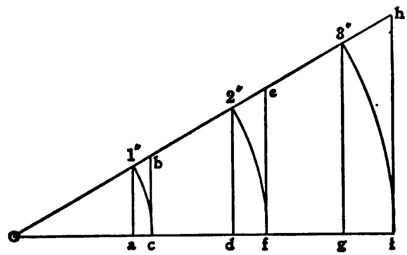


FIG. 1. — Triangles.

shows a 30-degree angle with 1, 2 and 3-inch arcs, 1 *c*, 2 *f*, and 3 *i*. It will be found that every similar measurement is in exact proportion to the radius, thus 2 *d* is exactly twice the length of 1 *a*, and *h i* is just three times *b c*. So, if we know the distance *a c* for a 1-inch radius for any angle, a similar distance as *g i* for the same angle will be in exact

proportion to the radius of the circle to one inch which is the base. All these parts are named as shown in Figs. 2, 3, and 4.

The exact proportions of all the various parts have been

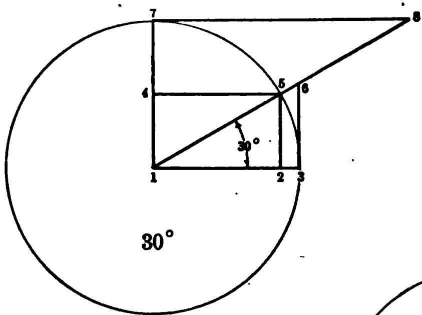


FIG. 2.

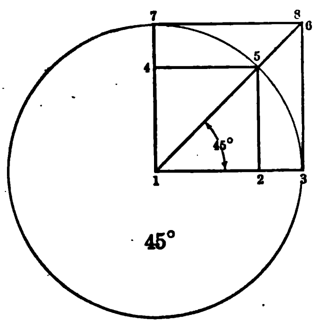


FIG. 3.

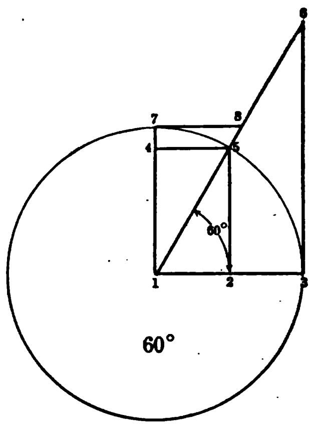


FIG. 4.

figured for each part of a degree that is likely to be needed in ordinary work, and these figures are given in the tables which follow. These numbers are simply multipliers or

constants for a radius of one, and for any other radius we multiply the numbers given by the radius we are using. These tables form the most accurate means of calculating many problems, as will be shown. These constants can represent *one* of anything, inches, feet, meters, or miles, and the answer will be in the same unit. In tool work they are usually in inches, but the relation is the same regardless of the unit.

Lines 1–3 and 1–7 are called radius of the circle.
1–2 is called the cosine of the angle.
4–5 is always the same as cosine of the angle.
2–3 is called the versed sine of the angle.
4–7 is called the co-versed sine of the angle.
2–5 is called the sine of the angle.
3–6 is called the tangent of the angle.
7–8 is called the co-tangent of the angle.
1–6 is called the secant of the angle.
1–8 is called the co-secant of the angle.

Angle is Always Taken each Side of the Center Line as Shown

The names always refer to the angle on one side of the center line and not to the total or included angle. In dealing with a 60-degree thread we divide this by a center line and call the angle 30 degrees in all our calculations. Everything is based on the radius of a circle, and a 1 radius is used as this base. Perhaps the three most important parts are the *sine*, the *tangent*, and the *secant*, these being 2–5, 3–6, and 1–6 in all three of the figures. From this it will be seen that the sine is half the chord, or the distance from the radius to the horizontal. The tangent 3–6 is the

distance from the horizontal radius to an extension of the radius at the angle given. The secant is the distance along the radius from the center to the tangent. From 2 to 3 is called the *versed sine*, and is the distance from the center of the chord to the outer circle.

The angle considered in this work is always less than 90 degrees, and the angle between the angle used and 90 degrees, or the angle which is necessary to complete this to 90 degrees, is called the complementary angle. In the first case the complementary angle is 60 degrees, in the second case 45 degrees, and in the third case 30 degrees. The *co-sine* is the distance 4–5, the *co-tangent* is 7–8, the *co-secant* is 1–8, and the *co-versed sine* is 4–7 in all three examples. In the 45-degree angle it will be seen that the various parts are alike in both angles, as the *sine* is the same as the cosine, while the sine of the angle of 30 degrees is the same as the cosine of the angle of 60 degrees. These facts will be borne out by the tables and can be seen by studying the diagrams or by making any calculation and then proving it as near as may be on the drawing board.

All this is interesting, but unless it is useful it has no value to the practical man, so we will see where it can be used to advantage in saving time and labor.

Perhaps the easiest application is in finding the depth of a V-thread without making any figures. The angle is 60 degrees or 30 degrees each side of the center line. The pitch is 1 inch so that each side is also an inch, and so the radius is an inch. The depth of the thread is the distance 1–2 or 4–5, and is the cosine of the angle. Looking in the table for the cosine of the angle of 30 degrees we find 0.86603, and as the radius is 1 this gives us the depth directly as

0.86603 inch. If the radius was 2 inches we would multiply by 2, or if it was ½ inch, divide by 2 and get the exact depth with almost no figuring. Suppose, on the other hand, that the thread was one inch deep and we want to find the length of one side, the angle remaining the same as before. In this case we have the depth which is the line 1–3, and we wish to find 1–6 which is the secant, so we look at the table again and find the secant of 30 degrees to be 1.1547 inches which is the length of the side.

Suppose you have a square bar 2½ inches on each side, what is the distance across the corners? Looking at the second example we see that the side of the square bar is represented by line 1–3 and the corner distance by the secant 1–6, so we look for the secant of 45 degrees (because we know that half the 90 degree angle of a square bar must be 45 degrees) and find 1.4142, which would be the distance if the bar was one inch square, so we multiply 1.4142 by 2½ and get 3.5355 inches as the distance across the corners, and can know that this is closer than we can measure, and is not a guess by any means.

Reversing this, we can find the side of a square that can be milled out of a round bar, such as the end of a reamer or tap. What square can we make on a 2-inch round reamer shank? The radius of the bar is the radius as 1–5 and the angle 45 degrees as before; half the side of the square will be the sine 2–5, which the table shows to be 0.70711, and as this is half the chord which makes the flat across the bar, we multiply this by 2 and get 1.41422 inches as the distance across the flats for a reamer shank of this size.

A very practical use for this kind of calculation is in spacing bolt holes or otherwise dividing a circle into any

number of equal parts. It is easy enough to get the length of each arc of the circumference by dividing 360 degrees by the number of divisions, but what we want is to find the chord or the distance from one point to the next in a straight line as a pair of dividers would step it off. First divide 360 by the number of divisions — say 9 — and get 40 degrees in each part. Fig. 5 shows this and we want the distance *a b* or the chord of the angle. This equals

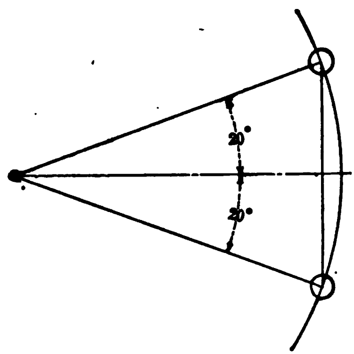


FIG. 5.—Spacing Bolt Circles.

twice the sine of half the angle. Half the angle is 20 degrees and the sine for this is .342. Twice this or 0.684 is the chord of the 40-degree angle for every inch of radius. If the circle is 14 inches in diameter the distance between the holes will be 7 times 0.684 or 4.788 inches. This is very quick and the most accurate method known.

Draftsmen often lay out jigs with the angles marked in degrees as in Fig. 6, overlooking the fact that the toolmaker has no convenient or accurate protractor for measuring the angle. Assume that a drawing shows three holes as *a*, *b*, and *c*, with

b and *c* 20 degrees apart. The distance from *a* to *b* is 3 inches, what is the distance from *b* to *c* or from *a* to *c*?

As the known radius is from *a* to *b*, the distance *b c* is the tangent of the angle, and the tangent for a 1-inch radius is .36397, so for a 3-inch radius it is $3 \times .36397 = 1.09191$ inches from *b* to *c* and at right angles to it.

But we need not depend on the accuracy of the square or of the way we use it, as we can find the distance from *a* to *c* just as easily and just as accurately as we did *b c*. This distance is the secant, and is 1.0642 for a 1-inch radius.

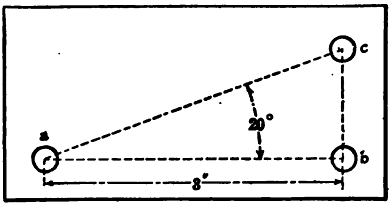


FIG. 6. — Laying out Jigs.

Multiplying this by $3 = 3.1926$ as the distance which can be accurately measured.

If the distance between *a* and *c* had been 3 inches, then *b c* would have been the sine and *a b* the cosine of the angle, both of which can be easily found from the tables.

It often happens that we want to find the angle of a roller or other piece of work as Fig. 7. Always work from the center line and continue the lines to complete the angle. Every triangle has the sides and they are called the "side opposite," "side adjacent," and "hypothenuse," the first being opposite the angle, the second the base line, and the third the slant line.

The following rules are very useful in this kind of work:

(1) Sine $= \dfrac{\text{Side Opp.}}{\text{Hypot.}}$

(2) Cosine $= \dfrac{\text{Side Adj.}}{\text{Hypot.}}$

(3) Tangent $= \dfrac{\text{Side Opp.}}{\text{Side Adj.}}$

(4) Co-Tangent $= \dfrac{\text{Side Adj.}}{\text{Side Opp.}}$

(5) Hypot. $= \dfrac{\text{Side Opp.}}{\text{Sine.}}$

(6) Side Opp. = Hypot. $\times$ Sine.

(7) Side Adj. = Hypot. $\times$ Cosine.

(8) Side Opp. = Side Adj. $\times$ Tangent.

(9) Side Adj. = Co-Tan. $\times$ Side Opp.

(10) Hypot. $= \dfrac{\text{Side Adj.}}{\text{Cosine.}}$

If we have the dimensions shown in Fig. 7, the side opposite, and the hypothenuse, we use formula No. 1, and dividing 2 by 4 we get $\frac{1}{2}$ or .5 as the sine of the angle. The table shows this to be the sine of the angle of 30 degrees, consequently this is a 30-degree angle.

If we have the side opposite and the side adjacent we use formula No. 3, and find that $\frac{2}{4} = \frac{1}{2}$ or .5 = the tangent of the angle. The table shows this to be the tangent of 26 degrees and 45 minutes.

Should it happen that we only knew the hypothenuse and the angle we use formula No. 6 and multiply 4 $\times$ .5 = 2, the side opposite. In the same way we can find the side adjacent by using formula No. 7. The cosine of 30 degrees in .866 and 4 $\times$ .866 = 3.464 inches as the side adjacent.

Having a bar of steel 2 by 3 inches, what is the distance across the corners? Either formulas 3 or 4 will answer for this. Taking No. 4 we have 2 as the side opposite, 3 as the side adjacent. Dividing 3 by 2 gives 1.5.

Looking under co-tangents for this we find 1.5108 after 33 degrees 30 minutes, which is nearly the correct angle. Then look for the secant of this and find 1.1958. Multiply this by 3 and get 3.5874 as the distance across the corners.

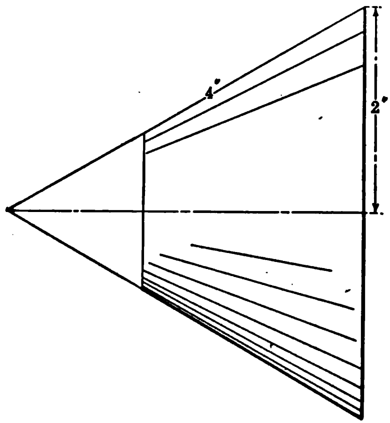


FIG. 7. — A Cone Roller.

The next two pages give all the transposition of these formula that will be needed for any case, and the tables which follow will give all the values of the different functions. A little practice will show how they save time and give accurate results.

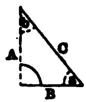
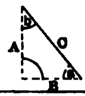
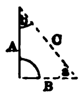
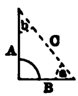
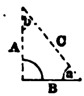
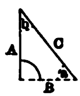
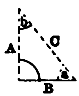
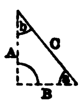
Full Lines ———— are given  Angle α		Dotted Lines ———— are to be found  Angle α α α.	
Figure	**Formulae**	**Figure**	**Formulae**
	$\cos a = \frac{B}{C}$ $\sin b = \frac{B}{C}$ $A = C \sin a$ $A = C \cos b$ $A = \sqrt{C^2 - B^2}$ $a = 90° - b°$ $b = 90° - a°$		$B = A \text{ cotg } a$ $C = \frac{A}{\sin a}$ $b = 90° - a°$
	$A = C \cos b$ $B = C \sin b$ $a = 90° - b°$		$B = A \text{ tang } a$ $C = \frac{A}{\cos b}$ $a = 90° - b°$
	$\text{tang } a = \frac{A}{B}$ $\text{tang } b = \frac{B}{A}$ $C = \frac{A}{\sin a}$ $C = \frac{A}{\cos b}$ $C = \sqrt{A^2 + B^2}$		$A = B \text{ tang } a$ $C = \frac{B}{\cos a}$ $b = 90° - a°$
	$\sin a = \frac{A}{C}$ $\cos b = \frac{A}{C}$ $B = C \sin b$ $B = C \cos A$ $B = \sqrt{C^2 - A^2}$ $a = 90° - b°$ $b = 90° - a°$		$A = B \text{ cotg } b$ $C = \frac{B}{\sin b}$ $a = 90° - b$
	Figure	**Formulae**	
		$A = C \sin a$ $B = C \cos a$ $b = 90° - a°$	

TABLE OF TRIGONOMETRICAL FORMULAS FOR RIGHT ANGLED TRIANGLES.

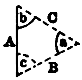
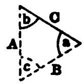
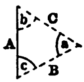
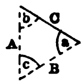
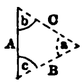
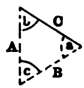
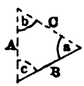
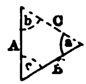
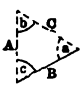
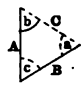
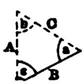
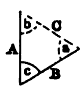
Figure	Formulae	Figure	Formulae
	$B = \frac{A \sin b}{\sin a}$ $C = \frac{A \sin (a+b)}{\sin a}$ $c = 180° - (a+b)°$ Contents $= F$ $F = \frac{A\ B \sin c}{2}$ $F = \frac{A \sin b \sin c}{2 \sin a}$		$A = \frac{C \sin a}{\sin (a+b)}$ $B = \frac{C \sin b}{\sin (a+b)}$ $c = 180° - (a+b)°$
	$B = \frac{A \sin (a+c)}{\sin a}$ $C = \frac{A \sin c}{\sin a}$ $b = 180° - (a+c)°$		$A = \frac{C \sin a}{\sin c}$ $B = \frac{C \sin (a+c)}{\sin c}$ $b = 180° - (a+c)°$
	$B = \frac{A \sin b}{\sin (b+c)}$ $C = \frac{A \sin c}{\sin (b+c)}$ $a = 180° - (b+c)°$		$A = \frac{C \sin (b+c)}{\sin c}$ $B = \frac{C \sin b}{\sin c}$ $a = 180° - (b+c)°$
	$A = \frac{B \sin a}{\sin b}$ $C = \frac{B \sin (a+b)}{\sin b}$ $c = 180° - (a+b)°$		$\sin b = \frac{B \sin a}{A}$ $c = 180° - (a+b)°$ $C = \frac{A \sin c}{\sin a}$ $F = \frac{A\ B \sin c}{2}$
	$A = \frac{B \sin (b+c)}{\sin b}$ $C = \frac{B \sin c}{\sin b}$ $a = 180° \ (b+c)°$		$\sin a = \frac{A \sin b}{B}$ $c = 180° - (a+b)°$ $C = \frac{A \sin c}{\sin a}$
	$A = \frac{B \sin a}{\sin (a-c)}$ $C = \frac{B \sin c}{\sin (a+c)}$ $b = 180° - (a+c)°$		$\mathrm{Tg}\ a = \frac{A \sin c}{B - A \cos c}$ $C = \sqrt{A^2 + B^2 - 2AB \cos c}$ $C = \sqrt{(A+B)^2 - 4AB \cos^2 \frac{c}{2}}$ $C = \sqrt{(A-B)^2 + 4AB \sin^2 \frac{c}{2}}$ $\mathrm{tg}\left(\frac{a-b}{2}\right) = \frac{A-B}{A+B} \mathrm{cotg} \frac{c}{2}$ $F = \frac{A\ B \sin c}{2}$

TABLE OF TRIGONOMETRICAL FORMULAS, COMMON PLANE TRIANGLE.

′	0°		1°		2°		3°		′
′	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	′
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.750	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.870	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.920	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00814	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04832	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	89°		88°		87°		86°		

′	4° Tan.	4° Co-tan.	5° Tan.	5° Co-tan.	6° Tan.	6° Co-tan.	7° Tan.	7° Co-tan.	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33154	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	9.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	85° Co-tan.	85° Tan.	84° Co-tan.	84° Tan.	83° Co-tan.	83° Tan.	82° Co-tan.	82° Tan.	′

	8°		9°		10°		11°		
′	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	7.01174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18383	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91235	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	81°		80°		79°		78°		

′	12° Tan.	12° Co-tan.	13° Tan.	13° Co-tan.	14° Tan.	14° Co-tan.	15° Tan.	15° Co-tan.	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4 61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27920	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28480	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03075	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	77°		76°		75°		74°		

′	16° Tan.	16° Co-tan.	17° Tan.	17° Co-tan.	18° Tan.	18° Co-tan.	19° Tan.	19° Co-tan.	′
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35019	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33160	3.01489	.35117	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94590	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	73° Co-tan.	73° Tan.	72° Co-tan.	72° Tan.	71° Co-tan.	71° Tan.	70° Co-tan.	70° Tan.	′

′	20°		21°		22°		23°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37124	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38862	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	69°		68°		67°		66°		

′	24° Tan.	24° Co-tan.	25° Tan.	25° Co-tan.	26° Tan.	26° Co-tan.	27° Tan.	27° Co-tan.	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45537	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97680	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52984	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Co-tan. 65°	Tan. 65°	Co-tan. 64°	Tan. 64°	Co-tan. 63°	Tan. 63°	Co-tan. 62°	Tan. 62°	′

	28°		29°		30°		31°		
	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	
0	.53171	1.88073	.55431	1.80405	.57735	1.73205	.60086	1.66428	60
1	.53208	1.87941	.55469	1.80281	.57774	1.73089	.60126	1.66318	59
2	.53246	1.87809	.55507	1.80158	.57813	1.72973	.60165	1.66209	58
3	.53283	1.87677	.55545	1.80034	.57851	1.72857	.60205	1.66099	57
4	.53320	1.87546	.55583	1.79911	.57890	1.72741	.60245	1.65990	56
5	.53358	1.87415	.55621	1.79788	.57929	1.72625	.60284	1.65881	55
6	.53395	1.87283	.55659	1.79665	.57968	1.72509	.60324	1.65772	54
7	.53432	1.87152	.55697	1.79542	.58007	1.72393	.60364	1.65663	53
8	.53470	1.87021	.55736	1.79419	.58046	1.72278	.60403	1.65534	52
9	.53507	1.86891	.55774	1.79296	.58085	1.72163	.60443	1.65445	51
10	.53545	1.86760	.55812	1.79174	.58124	1.72047	.60483	1.65337	50
11	.53582	1.86630	.55850	1.79051	.58162	1.71932	.60522	1.65228	49
12	.53620	1.86499	.55888	1.78929	.58201	1.71817	.60562	1.65120	48
13	.53657	1.86369	.55926	1.78807	.58240	1.71702	.60602	1.65011	47
14	.53694	1.86239	.55964	1.78685	.58279	1.71588	.60642	1.64903	46
15	.53732	1.86109	.56003	1.78563	.58318	1.71473	.60681	1.64795	45
16	.53769	1.85979	.56041	1.78441	.58357	1.71358	.60721	1.64687	44
17	.53807	1.85850	.56079	1.78319	.58396	1.71244	.60761	1.64579	43
18	.53844	1.85720	.56117	1.78198	.58435	1.71129	.60801	1.64471	42
19	.53882	1.85591	.56156	1.78077	.58474	1.71015	.60841	1.64363	41
20	.53920	1.85462	.56194	1.77955	.58513	1.70901	.60881	1.64256	40
21	.53957	1.85333	.56232	1.77834	.58552	1.70787	.60921	1.64148	39
22	.53995	1.85204	.56270	1.77713	.58591	1.70673	.60960	1.64041	38
23	.54032	1.85075	.56309	1.77592	.58631	1.70560	.61000	1.63934	37
24	.54070	1.84946	.56347	1.77471	.58670	1.70446	.61040	1.63826	36
25	.54107	1.84818	.56385	1.77351	.58709	1.70332	.61080	1.63719	35
26	.54145	1.84689	.56424	1.77230	.58748	1.70219	.61120	1.63612	34
27	.54183	1.84561	.56462	1.77110	.58787	1.70106	.61160	1.63505	33
28	.54220	1.84433	.56500	1.76990	.58826	1.69992	.61200	1.63398	32
29	.54258	1.84305	.56539	1.76869	.58865	1.69879	.61240	1.63292	31
30	.54296	1.84177	.56577	1.76749	.58904	1.69766	.61280	1.63185	30
31	.54333	1.84049	.56616	1.76630	.58944	1.69653	.61320	1.63079	29
32	.54371	1.83922	.56654	1.76510	.58983	1.69541	.61360	1.62972	28
33	.54409	1.83794	.56693	1.76390	.59022	1.69428	.61400	1.62866	27
34	.54446	1.83667	.56731	1.76271	.59061	1.69316	.61440	1.62760	26
35	.54484	1.83540	.56769	1.76151	.59101	1.69203	.61480	1.62654	25
36	.54522	1.83413	.56808	1.76032	.59140	1.69091	.61520	1.62548	24
37	.54560	1.83286	.56846	1.75913	.59179	1.68979	.61561	1.62442	23
38	.54597	1.83159	.56885	1.75794	.59218	1.68866	.61601	1.62336	22
39	.54635	1.83033	.56923	1.75675	.59258	1.68754	.61641	1.62230	21
40	.54673	1.82906	.56962	1.75556	.59297	1.68643	.61681	1.62125	20
41	.54711	1.82780	.57000	1.75437	.59336	1.68531	.61721	1.62019	19
42	.54748	1.82654	.57039	1.75319	.59376	1.68419	.61761	1.61914	18
43	.54786	1.82528	.57078	1.75200	.59415	1.68308	.61801	1.61808	17
44	.54824	1.82402	.57116	1.75082	.59454	1.68196	.61842	1.61703	16
45	.54862	1.82276	.57155	1.74964	.59494	1.68085	.61882	1.61598	15
46	.54900	1.82150	.57193	1.74846	.59533	1.67974	.61922	1.61493	14
47	.54938	1.82025	.57232	1.74728	.59573	1.67863	.61962	1.61388	13
48	.54975	1.81899	.57271	1.74610	.59612	1.67752	.62003	1.61283	12
49	.55013	1.81774	.57309	1.74492	.59651	1.67641	.62043	1.61179	11
50	.55051	1.81649	.57348	1.74375	.59691	1.67530	.62083	1.61074	10
51	.55089	1.81524	.57386	1.74257	.59730	1.67419	.62124	1.60970	9
52	.55127	1.81399	.57425	1.74140	.59770	1.67309	.62164	1.60865	8
53	.55165	1.81274	.57464	1.74022	.59809	1.67198	.62204	1.60761	7
54	.55203	1.81150	.57503	1.73905	.59849	1.67088	.62245	1.60657	6
55	.55241	1.81025	.57541	1.73788	.59888	1.66978	.62285	1.60553	5
56	.55279	1.80901	.57580	1.73671	.59928	1.66867	.62325	1.60449	4
57	.55317	1.80777	.57619	1.73555	.59967	1.66757	.62366	1.60345	3
58	.55355	1.80653	.57657	1.73438	.60007	1.66647	.62406	1.60241	2
59	.55393	1.80529	.57696	1.73321	.60046	1.66538	.62446	1.60137	1
60	.55431	1.80405	.57735	1.73205	.60086	1.66428	.62487	1.60033	0
	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	
	61°		60°		59°		58°		

′	32°		33°		34°		35°		′
	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	
0	[illegible]	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	[illegible]	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	[illegible]	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	[illegible]	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	[illegible]	1.59620	.65100	1.53595	.67620	1.47885	.70194	1.42462	56
5	[illegible]	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	[illegible]	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
[illegible]	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
[illegible]	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
[illegible]	[illegible]	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
[illegible]	[illegible]	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
[illegible]	[illegible]	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
[illegible]	[illegible]	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
[illegible]	[illegible]	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
[illegible]	[illegible]	1.53986	.67451	1.48256	.70021	1.42815	.72054	1.37638	0
	[illegible]	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	CO-TAN.	TAN.	′
	57°		56°		55°		54°		

′	36° Tan.	36° Co-tan.	37° Tan.	37° Co-tan.	38° Tan.	38° Co-tan.	39° Tan.	39° Co-tan.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74121	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan. 53°	Tan. 53°	Co-tan. 52°	Tan. 52°	Co-tan. 51°	Tan. 51°	Co-tan. 50°	Tan. 50°	′

′	32° Tan.	32° Co-tan.	33° Tan.	33° Co-tan.	34° Tan.	34° Co-tan.	35° Tan.	35° Co-tan.	′
0	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	60
1	.62527	1.59930	.64982	1.53888	.67493	1.48163	.70064	1.42726	59
2	.62568	1.59826	.65023	1.53791	.67536	1.48070	.70107	1.42638	58
3	.62608	1.59723	.65065	1.53693	.67578	1.47977	.70151	1.42550	57
4	.62649	1.59620	.65106	1.53595	.67620	1.47885	.70194	1.42462	56
5	.62689	1.59517	.65148	1.53497	.67663	1.47792	.70238	1.42374	55
6	.62730	1.59414	.65189	1.53400	.67705	1.47699	.70281	1.42286	54
7	.62770	1.59311	.65231	1.53302	.67748	1.47607	.70325	1.42198	53
8	.62811	1.59208	.65272	1.53205	.67790	1.47514	.70368	1.42110	52
9	.62852	1.59105	.65314	1.53107	.67832	1.47422	.70412	1.42022	51
10	.62892	1.59002	.65355	1.53010	.67875	1.47330	.70455	1.41934	50
11	.62933	1.58900	.65397	1.52913	.67917	1.47238	.70499	1.41847	49
12	.62973	1.58797	.65438	1.52816	.67960	1.47146	.70542	1.41759	48
13	.63014	1.58695	.65480	1.52719	.68002	1.47053	.70586	1.41672	47
14	.63055	1.58593	.65521	1.52622	.68045	1.46962	.70629	1.41584	46
15	.63095	1.58490	.65563	1.52525	.68088	1.46870	.70673	1.41497	45
16	.63136	1.58388	.65604	1.52429	.68130	1.46778	.70717	1.41409	44
17	.63177	1.58286	.65646	1.52332	.68173	1.46686	.70760	1.41322	43
18	.63217	1.58184	.65688	1.52235	.68215	1.46595	.70804	1.41235	42
19	.63258	1.58083	.65729	1.52139	.68258	1.46503	.70848	1.41148	41
20	.63299	1.57981	.65771	1.52043	.68301	1.46411	.70891	1.41061	40
21	.63340	1.57879	.65813	1.51946	.68343	1.46320	.70935	1.40974	39
22	.63380	1.57778	.65854	1.51850	.68386	1.46229	.70979	1.40887	38
23	.63421	1.57676	.65896	1.51754	.68429	1.46137	.71023	1.40800	37
24	.63462	1.57575	.65938	1.51658	.68471	1.46046	.71066	1.40714	36
25	.63503	1.57474	.65980	1.51562	.68514	1.45955	.71110	1.40627	35
26	.63544	1.57372	.66021	1.51466	.68557	1.45864	.71154	1.40540	34
27	.63584	1.57271	.66063	1.51370	.68600	1.45773	.71198	1.40454	33
28	.63625	1.57170	.66105	1.51275	.68642	1.45682	.71242	1.40367	32
29	.63666	1.57069	.66147	1.51179	.68685	1.45592	.71285	1.40281	31
30	.63707	1.56969	.66189	1.51084	.68728	1.45501	.71329	1.40195	30
31	.63748	1.56868	.66230	1.50988	.68771	1.45410	.71373	1.40109	29
32	.63789	1.56767	.66272	1.50893	.68814	1.45320	.71417	1.40022	28
33	.63830	1.56667	.66314	1.50797	.68857	1.45229	.71461	1.39936	27
34	.63871	1.56566	.66356	1.50702	.68900	1.45139	.71505	1.39850	26
35	.63912	1.56466	.66398	1.50607	.68942	1.45049	.71549	1.39764	25
36	.63953	1.56366	.66440	1.50512	.68985	1.44958	.71593	1.39679	24
37	.63994	1.56265	.66482	1.50417	.69028	1.44868	.71637	1.39593	23
38	.64035	1.56165	.66524	1.50322	.69071	1.44778	.71681	1.39507	22
39	.64076	1.56065	.66566	1.50228	.69114	1.44688	.71725	1.39421	21
40	.64117	1.55966	.66608	1.50133	.69157	1.44598	.71769	1.39336	20
41	.64158	1.55866	.66650	1.50038	.69200	1.44508	.71813	1.39250	19
42	.64199	1.55766	.66692	1.49944	.69243	1.44418	.71857	1.39165	18
43	.64240	1.55666	.66734	1.49849	.69286	1.44329	.71901	1.39079	17
44	.64281	1.55567	.66776	1.49755	.69329	1.44239	.71946	1.38994	16
45	.64322	1.55467	.66818	1.49661	.69372	1.44149	.71990	1.38909	15
46	.64363	1.55368	.66860	1.49566	.69416	1.44060	.72034	1.38824	14
47	.64404	1.55269	.66902	1.49472	.69459	1.43970	.72078	1.38738	13
48	.64446	1.55170	.66944	1.49378	.69502	1.43881	.72122	1.38653	12
49	.64487	1.55071	.66986	1.49284	.69545	1.43792	.72166	1.38568	11
50	.64528	1.54972	.67028	1.49190	.69588	1.43703	.72211	1.38484	10
51	.64569	1.54873	.67071	1.49097	.69631	1.43614	.72255	1.38399	9
52	.64610	1.54774	.67113	1.49003	.69675	1.43525	.72299	1.38314	8
53	.64652	1.54675	.67155	1.48909	.69718	1.43436	.72344	1.38229	7
54	.64693	1.54576	.67197	1.48816	.69761	1.43347	.72388	1.38145	6
55	.64734	1.54478	.67239	1.48722	.69804	1.43258	.72432	1.38060	5
56	.64775	1.54379	.67282	1.48629	.69847	1.43169	.72477	1.37976	4
57	.64817	1.54281	.67324	1.48536	.69891	1.43080	.72521	1.37891	3
58	.64858	1.54183	.67366	1.48442	.69934	1.42992	.72565	1.37807	2
59	.64899	1.54085	.67409	1.48349	.69977	1.42903	.72610	1.37722	1
60	.64941	1.53986	.67451	1.48256	.70021	1.42815	.72654	1.37638	0
′	57° Co-tan.	57° Tan.	56° Co-tan.	56° Tan.	55° Co-tan.	55° Tan.	54° Co-tan.	54° Tan.	′

′	36° Tan.	36° Co-tan.	37° Tan.	37° Co-tan.	38° Tan.	38° Co-tan.	39° Tan.	39° Co-tan.	′
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36133	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74121	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	53°		52°		51°		50°		

′	40° Tan.	40° Co-tan.	41° Tan.	41° Co-tan.	42° Tan.	42° Co-tan.	43° Tan.	43° Co-tan.	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85307	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	Co-tan.	Tan.	′
	49°		48°		47°		46°		

′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′	′	44° Tan.	44° Co-tan.	′
0	.96569	1.03553	60	21	.97756	1.02295	39	41	.98901	1.01112	19
1	.96625	1.03493	59	22	.97813	1.02236	38	42	.98958	1.01053	18
2	.96681	1.03433	58	23	.97870	1.02176	37	43	.99016	1.00994	17
3	.96738	1.03372	57	24	.97927	1.02117	36	44	.99073	1.00935	16
4	.96794	1.03312	56	25	.97984	1.02057	35	45	.99131	1.00876	15
5	.96850	1.03252	55	26	.98041	1.01998	34	46	.99189	1.00818	14
6	.96907	1.03192	54	27	.98098	1.01939	33	47	.99247	1.00759	13
7	.96963	1.03132	53	28	.98155	1.01879	32	48	.99304	1.00701	12
8	.97020	1.03072	52	29	.98213	1.01820	31	49	.99362	1.00642	11
9	.97076	1.03012	51	30	.98270	1.01761	30	50	.99420	1.00583	10
10	.97133	1.02952	50	31	.98327	1.01702	29	51	.99478	1.00525	9
11	.97189	1.02892	49	32	.98384	1.01642	28	52	.99536	1.00467	8
12	.97246	1.02832	48	33	.98441	1.01583	27	53	.99594	1.00408	7
13	.97302	1.02772	47	34	.98499	1.01524	26	54	.99652	1.00350	6
14	.97359	1.02713	46	35	.98556	1.01465	25	55	.99710	1.00291	5
15	.97416	1.02653	45	36	.98613	1.01406	24	56	.99768	1.00233	4
16	.97472	1.02593	44	37	.98671	1.01347	23	57	.99826	1.00175	3
17	.97529	1.02533	43	38	.98728	1.01288	22	58	.99884	1.00116	2
18	.97586	1.02474	42	39	.98786	1.01229	21	59	.99942	1.00058	1
19	.97643	1.02414	41	40	.98843	1.01170	20	60	1	1	0
20	.97700	1.02355	40								
′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′	′	Co-tan. 45°	Tan. 45°	′

NATURAL SINES AND COSINES

′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′	′	0° Sine	0° Cosine	′
0	.00000	1	60	21	.00611	.99998	39	41	.01193	.99993	19
1	.00029	1	59	22	.00640	.99998	38	42	.01222	.99993	18
2	.00058	1	58	23	.00669	.99998	37	43	.01251	.99992	17
3	.00087	1	57	24	.00698	.99998	36	44	.01280	.99992	16
4	.00116	1	56	25	.00727	.99997	35	45	.01309	.99991	15
5	.00145	1	55	26	.00756	.99997	34	46	.01338	.99991	14
6	.00175	1	54	27	.00785	.99997	33	47	.01367	.99991	13
7	.00204	1	53	28	.00814	.99997	32	48	.01396	.99990	12
8	.00233	1	52	29	.00844	.99996	31	49	.01425	.99990	11
9	.00262	1	51	30	.00873	.99996	30	50	.01454	.99989	10
10	.00291	1	50	31	.00902	.99996	29	51	.01483	.99989	9
11	.00320	.99999	49	32	.00931	.99996	28	52	.01513	.99989	8
12	.00349	.99999	48	33	.00960	.99995	27	53	.01542	.99988	7
13	.00378	.99999	47	34	.00989	.99995	26	54	.01571	.99988	6
14	.00407	.99999	46	35	.01018	.99995	25	55	.01600	.99987	5
15	.00436	.99999	45	36	.01047	.99995	24	56	.01629	.99987	4
16	.00465	.99999	44	37	.01076	.99994	23	57	.01658	.99986	3
17	.00495	.99999	43	38	.01105	.99994	22	58	.01687	.99986	2
18	.00524	.99999	42	39	.01134	.99994	21	59	.01716	.99985	1
19	.00553	.99998	41	40	.01164	.99993	20	60	.01745	.99985	0
20	.00582	.99998	40								
′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′	′	Cosine 89°	Sine 89°	′

′	1° Sine	1° Cosine	2° Sine	2° Cosine	3° Sine	3° Cosine	4° Sine	4° Cosine	′
0	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine 88°	Sine 88°	Cosine 87°	Sine 87°	Cosine 86°	Sine 86°	Cosine 85°	Sine 85°	′

′	5° Sine	5° Cosine	6° Sine	6° Cosine	7° Sine	7° Cosine	8° Sine	8° Cosine	′
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		

′	9° Sine	9° Cosine	10° Sine	10° Cosine	11° Sine	11° Cosine	12° Sine	12° Cosine	′
0	.15643	.98769	.17365	.98481	.19081	.98163	.20791	.97815	60
1	.15672	.98764	.17393	.98476	.19109	.98157	.20820	.97809	59
2	.15701	.98760	.17422	.98471	.19138	.98152	.20848	.97803	58
3	.15730	.98755	.17451	.98466	.19167	.98146	.20877	.97797	57
4	.15758	.98751	.17479	.98461	.19195	.98140	.20905	.97791	56
5	.15787	.98746	.17508	.98455	.19224	.98135	.20933	.97784	55
6	.15816	.98741	.17537	.98450	.19252	.98129	.20962	.97778	54
7	.15845	.98737	.17565	.98445	.19281	.98124	.20990	.97772	53
8	.15873	.98732	.17594	.98440	.19309	.98118	.21019	.97766	52
9	.15902	.98728	.17623	.98435	.19338	.98112	.21047	.97760	51
10	.15931	.98723	.17651	.98430	.19366	.98107	.21076	.97754	50
11	.15959	.98718	.17680	.98425	.19395	.98101	.21104	.97748	49
12	.15988	.98714	.17708	.98420	.19423	.98096	.21132	.97742	48
13	.16017	.98709	.17737	.98414	.19452	.98090	.21161	.97735	47
14	.16046	.98704	.17766	.98409	.19481	.98084	.21189	.97729	46
15	.16074	.98700	.17794	.98404	.19509	.98079	.21218	.97723	45
16	.16103	.98695	.17823	.98399	.19538	.98073	.21246	.97717	44
17	.16132	.98690	.17852	.98394	.19566	.98067	.21275	.97711	43
18	.16160	.98689	.17880	.98389	.19595	.98061	.21303	.97705	42
19	.16189	.98681	.17909	.98383	.19623	.98056	.21331	.97698	41
20	.16218	.98676	.17937	.98378	.19652	.98050	.21360	.97692	40
21	.16246	.98671	.17966	.98373	.19680	.98044	.21388	.97686	39
22	.16275	.98667	.17995	.98368	.19709	.98039	.21417	.97680	38
23	.16304	.98662	.18023	.98362	.19737	.98033	.21445	.97673	37
24	.16333	.98657	.18052	.98357	.19766	.98027	.21474	.97667	36
25	.16361	.98652	.18081	.98352	.19794	.98021	.21502	.97661	35
26	.16390	.98648	.18109	.98347	.19823	.98016	.21530	.97655	34
27	.16419	.98643	.18138	.98341	.19851	.98010	.21559	.97648	33
28	.16447	.98638	.18166	.98336	.19880	.98004	.21587	.97642	32
29	.16476	.98633	.18195	.98331	.19908	.97988	.21616	.97636	31
30	.16505	.98629	.18224	.98325	.19937	.97992	.21644	.97630	30
31	.16533	.98624	.18252	.98320	.19965	.97987	.21672	.97623	29
32	.16562	.98619	.18281	.98315	.19994	.97981	.21701	.97617	28
33	.16591	.98614	.18309	.98310	.20022	.97975	.21729	.97611	27
34	.16620	.98609	.18338	.98304	.20031	.97969	.21758	.97604	26
35	.16648	.98604	.18367	.98299	.20079	.97963	.21786	.97598	25
36	.16677	.98600	.18395	.98294	.20108	.97958	.21814	.97592	24
37	.16706	.98595	.18424	.98288	.20136	.97952	.21843	.97585	23
38	.16734	.98590	.18452	.98283	.20165	.97946	.21871	.97579	22
39	.16763	.98585	.18481	.98277	.20193	.97940	.21899	.97573	21
40	.16792	.98580	.18509	.98272	.20222	.97934	.21928	.97566	20
41	.16820	.98575	.18538	.98267	.20250	.97928	.21956	.97560	19
42	.16849	.98570	.18567	.98261	.20279	.97922	.21985	.97553	18
43	.16878	.98565	.18595	.98256	.20307	.97916	.22013	.97547	17
44	.16906	.98561	.18624	.98250	.20336	.97910	.22041	.97541	16
45	.16935	.98556	.18652	.98245	.20364	.97905	.22070	.97534	15
46	.16964	.98551	.18681	.98240	.20393	.97899	.22098	.97528	14
47	.16992	.98546	.18710	.98234	.20421	.97893	.22126	.97521	13
48	.17021	.98541	.18738	.98229	.20450	.97887	.22155	.97515	12
49	.17050	.98536	.18767	.98223	.20478	.97881	.22183	.97508	11
50	.17078	.98531	.18795	.98218	.20507	.97875	.22212	.97502	10
51	.17107	.98526	.18824	.98212	.20535	.97869	.22240	.97496	9
52	.17136	.98521	.18852	.98207	.20563	.97863	.22268	.97489	8
53	.17164	.98516	.18881	.98201	.20592	.97857	.22297	.97483	7
54	.17193	.98511	.18910	.98196	.20620	.97851	.22325	.97476	6
55	.17222	.98506	.18938	.98190	.20649	.97845	.22353	.97470	5
56	.17250	.98501	.18967	.98185	.20677	.97839	.22382	.97463	4
57	.17279	.98496	.18995	.98179	.20706	.97833	.22410	.97457	3
58	.17308	.98491	.19024	.98174	.20734	.97827	.22438	.97450	2
59	.17336	.98486	.19052	.98168	.20763	.97821	.22467	.97444	1
60	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	0
′	80° Cosine	80° Sine	79° Cosine	79° Sine	78° Cosine	78° Sine	77° Cosine	77° Sine	′

′	13° Sine	13° Cosine	14° Sine	14° Cosine	15° Sine	15° Cosine	16° Sine	16° Cosine	′
0	.22495	.97437	.24192	.97030	.25882	.96593	.27564	.96126	60
1	.22523	.97430	.24220	.97023	.25910	.96585	.27592	.96118	59
2	.22552	.97424	.24249	.97015	.25938	.96578	.27620	.96110	58
3	.22580	.97417	.24277	.97008	.25966	.96570	.27648	.96102	57
4	.22608	.97411	.24305	.97001	.25994	.96562	.27676	.96094	56
5	.22637	.97404	.24333	.96994	.26022	.96555	.27704	.96086	55
6	.22665	.97398	.24362	.96987	.26050	.96547	.27731	.96078	54
7	.22693	.97391	.24390	.96980	.26079	.96540	.27759	.96070	53
8	.22722	.97384	.24418	.96973	.26107	.96532	.27787	.96062	52
9	.22750	.97378	.24446	.96966	.26135	.96524	.27815	.96054	51
10	.22778	.97371	.24474	.96959	.26163	.96517	.27843	.96046	50
11	.22807	.97365	.24503	.96952	.26191	.96509	.27871	.96037	49
12	.22835	.97358	.24531	.96945	.26219	.96502	.27899	.96029	48
13	.22863	.97351	.24559	.96937	.26247	.96494	.27927	.96021	47
14	.22892	.97345	.24587	.96930	.26275	.96486	.27955	.96013	46
15	.22920	.97338	.24615	.96923	.26303	.96479	.27983	.96005	45
16	.22948	.97331	.24644	.96916	.26331	.96471	.28011	.95997	44
17	.22977	.97325	.24672	.96909	.26359	.96463	.28039	.95989	43
18	.23005	.97318	.24700	.96902	.26387	.96456	.28067	.95981	42
19	.23033	.97311	.24728	.96894	.26415	.96448	.28095	.95972	41
20	.23062	.97304	.24756	.96887	.26443	.96440	.28123	.95964	40
21	.23090	.97298	.24784	.96880	.26471	.96433	.28150	.95956	39
22	.23118	.97291	.24813	.96873	.26500	.96425	.28178	.95948	38
23	.23146	.97284	.24841	.96866	.26528	.96417	.28206	.95940	37
24	.23175	.97278	.24869	.96858	.26556	.96410	.28234	.95931	36
25	.23203	.97271	.24897	.96851	.26584	.96402	.28262	.95923	35
26	.23231	.97264	.24925	.96844	.26612	.96394	.28290	.95915	34
27	.23260	.97257	.24954	.96837	.26640	.96386	.28318	.95907	33
28	.23288	.97251	.24982	.96829	.26668	.96379	.28346	.95898	32
29	.23316	.97244	.25010	.96822	.26696	.96371	.28374	.95890	31
30	.23345	.97237	.25038	.96815	.26724	.96363	.28402	.95882	30
31	.23373	.97230	.25066	.96807	.26752	.96355	.28429	.95874	29
32	.23401	.97223	.25094	.96800	.26780	.96347	.28457	.95865	28
33	.23429	.97217	.25122	.96793	.26808	.96340	.28485	.95857	27
34	.23458	.97210	.25151	.96786	.26836	.96332	.28513	.95849	26
35	.23486	.97203	.25179	.96778	.26864	.96324	.28541	.95841	25
36	.23514	.97196	.25207	.96771	.26892	.96316	.28569	.95832	24
37	.23542	.97189	.25235	.96764	.26920	.96308	.28597	.95824	23
38	.23571	.97182	.25263	.96756	.26948	.96301	.28625	.95816	22
39	.23599	.97176	.25291	.96749	.26976	.96293	.28652	.95807	21
40	.23627	.97169	.25320	.96742	.27004	.96285	.28680	.95799	20
41	.23656	.97162	.25348	.96734	.27032	.96277	.28708	.95791	19
42	.23684	.97155	.25376	.96727	.27060	.96269	.28736	.95782	18
43	.23712	.97148	.25404	.96719	.27088	.96261	.28764	.95774	17
44	.23740	.97141	.25432	.96712	.27116	.96253	.28792	.95766	16
45	.23769	.97134	.25460	.96705	.27144	.96246	.28820	.95757	15
46	.23797	.97127	.25488	.96697	.27172	.96238	.28847	.95749	14
47	.23825	.97120	.25516	.96690	.27200	.96230	.28875	.95740	13
48	.23853	.97113	.25545	.96682	.27228	.96222	.28903	.95732	12
49	.23882	.97106	.25573	.96675	.27256	.96214	.28931	.95724	11
50	.23910	.97100	.25601	.96667	.27284	.96206	.28959	.95715	10
51	.23938	.97093	.25629	.96660	.27312	.96198	.28987	.95707	9
52	.23966	.97086	.25657	.96653	.27340	.96190	.29015	.95698	8
53	.23995	.97079	.25685	.96645	.27368	.96182	.29042	.95690	7
54	.24023	.97072	.25713	.96638	.27396	.96174	.29070	.95681	6
55	.24051	.97065	.25741	.96630	.27424	.96166	.29098	.95673	5
56	.24079	.97058	.25769	.96623	.27452	.96158	.29126	.95664	4
57	.24108	.97051	.25798	.96615	.27480	.96150	.29154	.95656	3
58	.24136	.97044	.25826	.96608	.27508	.96142	.29182	.95647	2
59	.24164	.97037	.25854	.96600	.27536	.96134	.29209	.95639	1
60	.24192	.97030	.25882	.96593	.27564	.96126	.29237	.95630	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	76°		75°		74°		73°		

′	17°		18°		19°		20°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	60
1	.29265	.95622	.30929	.95097	.32584	.94542	.34229	.93959	59
2	.29293	.95613	.30957	.95088	.32612	.94533	.34257	.93949	58
3	.29321	.95605	.30985	.95079	.32639	.94523	.34284	.93939	57
4	.29348	.95596	.31012	.95070	.32667	.94514	.34311	.93929	56
5	.29376	.95588	.31040	.95061	.32694	.94504	.34339	.93919	55
6	.29404	.95579	.31068	.95052	.32722	.94495	.34366	.93909	54
7	.29432	.95571	.31095	.95043	.32749	.94485	.34393	.93899	53
8	.29460	.95562	.31123	.95033	.32777	.94476	.34421	.93889	52
9	.29487	.95554	.31151	.95024	.32804	.94466	.34448	.93879	51
10	.29515	.95545	.31178	.95015	.32832	.94457	.34475	.93869	50
11	.29543	.95536	.31206	.95006	.32859	.94447	.34503	.93859	49
12	.29571	.95528	.31233	.94997	.32887	.94438	.34530	.93849	48
13	.29599	.95519	.31261	.94988	.32914	.94428	.34557	.93839	47
14	.29626	.95511	.31289	.94979	.32942	.94418	.34584	.93829	46
15	.29654	.95502	.31316	.94970	.32969	.94409	.34612	.93819	45
16	.29682	.95493	.31344	.94961	.32997	.94399	.34639	.93809	44
17	.29710	.95485	.31372	.94952	.33024	.94390	.34666	.93799	43
18	.29737	.95476	.31399	.94943	.33051	.94380	.34694	.93789	42
19	.29765	.95467	.31427	.94933	.33079	.94370	.34721	.93779	41
20	.29793	.95459	.31454	.94924	.33106	.94361	.34748	.93769	40
21	.29821	.95450	.31482	.94915	.33134	.94351	.34775	.93759	39
22	.29849	.95441	.31510	.94906	.33161	.94342	.34803	.93748	38
23	.29876	.95433	.31537	.94897	.33189	.94332	.34830	.93738	37
24	.29904	.95424	.31565	.94888	.33216	.94322	.34857	.93728	36
25	.29932	.95415	.31593	.94878	.33244	.94313	.34884	.93718	35
26	.29960	.95407	.31620	.94869	.33271	.94303	.34912	.93708	34
27	.29987	.95398	.31648	.94860	.33298	.94293	.34939	.93698	33
28	.30015	.95389	.31675	.94851	.33326	.94284	.34966	.93688	32
29	.30043	.95380	.31703	.94842	.33353	.94274	.34993	.93677	31
30	.30071	.95372	.31730	.94832	.33381	.94264	.35021	.93667	30
31	.30098	.95363	.31758	.94823	.33408	.94254	.35048	.93657	29
32	.30126	.95354	.31786	.94814	.33436	.94245	.35075	.93647	28
33	.30154	.95345	.31813	.94805	.33463	.94235	.35102	.93637	27
34	.30182	.95337	.31841	.94795	.33490	.94225	.35130	.93626	26
35	.30209	.95328	.31868	.94786	.33518	.94215	.35157	.93616	25
36	.30237	.95319	.31896	.94777	.33545	.94206	.35184	.93606	24
37	.30265	.95310	.31923	.94768	.33573	.94196	.35211	.93596	23
38	.30292	.95301	.31951	.94758	.33600	.94186	.35239	.93585	22
39	.30320	.95293	.31979	.94749	.33627	.94176	.35266	.93575	21
40	.30348	.95284	.32006	.94740	.33655	.94167	.35293	.93565	20
41	.30376	.95275	.32034	.94730	.33682	.94157	.35320	.93555	19
42	.30403	.95266	.32061	.94721	.33710	.94147	.35347	.93544	18
43	.30431	.95257	.32089	.94712	.33737	.94137	.35375	.93534	17
44	.30459	.95248	.32116	.94702	.33764	.94127	.35402	.93524	16
45	.30486	.95240	.32144	.94693	.33792	.94118	.35429	.93514	15
46	.30514	.95231	.32171	.94684	.33819	.94108	.35456	.93503	14
47	.30542	.95222	.32199	.94674	.33846	.94098	.35484	.93493	13
48	.30570	.95213	.32227	.94665	.33874	.94088	.35511	.93483	12
49	.30597	.95204	.32254	.94656	.33901	.94078	.35538	.93472	11
50	.30625	.95195	.32282	.94646	.33929	.94068	.35565	.93462	10
51	.30653	.95186	.32309	.94637	.33956	.94058	.35592	.93452	9
52	.30680	.95177	.32337	.94627	.33983	.94049	.35619	.93441	8
53	.30708	.95168	.32364	.94618	.34011	.94039	.35647	.93431	7
54	.30736	.95159	.32392	.94609	.34038	.94029	.35674	.93420	6
55	.30763	.95150	.32419	.94599	.34065	.94019	.35701	.93410	5
56	.30791	.95142	.32447	.94590	.34093	.94009	.35728	.93400	4
57	.30819	.95133	.32474	.94580	.34120	.93999	.35755	.93389	3
58	.30846	.95124	.32502	.94571	.34147	.93989	.35782	.93379	2
59	.30874	.95115	.32529	.94561	.34175	.93979	.35810	.93368	1
60	.30902	.95106	.32557	.94552	.34202	.93969	.35837	.93358	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	72°		71°		70°		69°		

′	21° Sine	21° Cosine	22° Sine	22° Cosine	23° Sine	23° Cosine	24° Sine	24° Cosine	′
0	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine 68°	Sine 68°	Cosine 67°	Sine 67°	Cosine 66°	Sine 66°	Cosine 65°	Sine 65°	′

′	25° Sine	25° Cosine	26° Sine	26° Cosine	27° Sine	27° Cosine	28° Sine	28° Cosine	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	9
52	.43628	.89981	45192	.89206	.46742	.88404	.48277	.87575	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		

′	29° Sine	29° Cosine	30° Sine	30° Cosine	31° Sine	31° Cosine	32° Sine	32° Cosine	′
0	.48481	.87462	.50000	.86603	.51504	.85717	.52992	.84805	60
1	.48506	.87448	.50025	.86588	.51529	.85702	.53017	.84789	59
2	.48532	.87434	.50050	.86573	.51554	.85687	.53041	.84774	58
3	.48557	.87420	.50076	.86559	.51579	.85672	.53066	.84759	57
4	.48583	.87406	.50101	.86544	.51604	.85657	.53091	.84743	56
5	.48608	.87391	.50126	.86530	.51628	.85642	.53115	.84728	55
6	.48634	.87377	.50151	.86515	.51653	.85627	.53140	.84712	54
7	.48659	.87363	.50176	.86501	.51678	.85612	.53164	.84697	53
8	.48684	.87349	.50201	.86486	.51703	.85597	.53189	.84681	52
9	.48710	.87335	.50227	.86471	.51728	.85582	.53214	.84666	51
10	.48735	.87321	.50252	.86457	.51753	.85567	.53238	.84650	50
11	.48761	.87306	.50277	.86442	.51778	.85551	.53263	.84635	49
12	.48786	.87292	.50302	.86427	.51803	.85536	.53288	.84619	48
13	.48811	.87278	.50327	.86413	.51828	.85521	.53312	.84604	47
14	.48837	.87264	.50352	.86398	.51852	.85506	.53337	.84588	46
15	.48862	.87250	.50377	.86384	.51877	.85491	.53361	.84573	45
16	.48888	.87235	.50403	.86369	.51902	.85476	.53386	.84557	44
17	.48913	.87221	.50428	.86354	.51927	.85461	.53411	.84542	43
18	.48938	.87207	.50453	.86340	.51952	.85446	.53435	.84526	42
19	.48964	.87193	.50478	.86325	.51977	.85431	.53460	.84511	41
20	.48989	.87178	.50503	.86310	.52002	.85416	.53484	.84495	40
21	.49014	.87164	.50528	.86295	.52026	.85401	.53509	.84480	39
22	.49040	.87150	.50553	.86281	.52051	.85385	.53534	.84464	38
23	.49065	.87136	.50578	.86266	.52076	.85370	.53558	.84448	37
24	.49090	.87121	.50603	.86251	.52101	.85355	.53583	.84433	36
25	.49116	.87107	.50628	.86237	.52126	.85340	.53607	.84417	35
26	.49141	.87093	.50654	.86222	.52151	.85325	.53632	.84402	34
27	.49166	.87079	.50679	.86207	.52175	.85310	.53656	.84386	33
28	.49192	.87064	.50704	.86192	.52200	.85294	.53681	.84370	32
29	.49217	.87050	.50729	.86178	.52225	.85279	.53705	.84355	31
30	.49242	.87036	.50754	.86163	.52250	.85264	.53730	.84339	30
31	.49268	.87021	.50779	.86148	.52275	.85249	.53754	.84324	29
32	.49293	.87007	.50804	.86133	.52299	.85234	.53779	.84308	28
33	.49318	.86993	.50829	.86119	.52324	.85218	.53804	.84292	27
34	.49344	.86978	.50854	.86104	.52349	.85203	.53828	.84277	26
35	.49369	.86964	.50879	.86089	.52374	.85188	.53853	.84261	25
36	.49394	.86949	.50904	.86074	.52399	.85173	.53877	.84245	24
37	.49419	.86935	.50929	.86059	.52423	.85157	.53902	.84230	23
38	.49445	.86921	.50954	.86045	.52448	.85142	.53926	.84214	22
39	.49470	.86906	.50979	.86030	.52473	.85127	.53951	.84198	21
40	.49495	.86892	.51004	.86015	.52498	.85112	.53975	.84182	20
41	.49521	.86878	.51029	.86000	.52522	.85096	.54000	.84167	19
42	.49546	.86863	.51054	.85985	.52547	.85081	.54024	.84151	18
43	.49571	.86849	.51079	.85970	.52572	.85066	.54049	.84135	17
44	.49596	.86834	.51104	.85956	.52597	.85051	.54073	.84120	16
45	.49622	.86820	.51129	.85941	.52621	.85035	.54097	.84104	15
46	.49647	.86805	.51154	.85926	.52646	.85020	.54122	.84088	14
47	.49672	.86791	.51179	.85911	.52671	.85005	.54146	.84072	13
48	.49697	.86777	.51204	.85896	.52696	.84989	.54171	.84057	12
49	.49723	.86762	.51229	.85881	.52720	.84974	.54195	.84041	11
50	.49748	.86748	.51254	.85866	.52745	.84959	.54220	.84025	10
51	.49773	.86733	.51279	.85851	.52770	.84943	.54244	.84009	9
52	.49798	.86719	.51304	.85836	.52794	.84928	.54269	.83994	8
53	.49824	.86704	.51329	.85821	.52819	.84913	.54293	.83978	7
54	.49849	.86690	.51354	.85806	.52844	.84897	.54317	.83962	6
55	.49874	.86675	.51379	.85792	.52869	.84882	.54342	.83946	5
56	.49899	.86661	.51404	.85777	.52893	.84866	.54366	.83930	4
57	.49924	.86646	.51429	.85762	.52918	.84851	.54391	.83915	3
58	.49950	.86632	.51454	.85747	.52943	.84836	.54415	.83899	2
59	.49975	.86617	.51479	.85732	.52967	.84820	.54440	.83883	1
60	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	0
′	Cosine	Sine 60°	Cosine	Sine 59°	Cosine	Sine 58°	Cosine	Sine 57°	′

′	33° Sine	33° Cosine	34° Sine	34° Cosine	35° Sine	35° Cosine	36° Sine	36° Cosine	′
0	.54464	.83867	.55919	.82904	.57358	.81915	.58779	.80902	60
1	.54488	.83851	.55943	.82887	.57381	.81899	.58802	.80885	59
2	.54513	.83835	.55968	.82871	.57405	.81882	.58826	.80867	58
3	.54537	.83819	.55992	.82855	.57429	.81865	.58849	.80850	57
4	.54561	.83804	.56016	.82839	.57453	.81848	.58873	.80833	56
5	.54586	.83788	.56040	.82822	.57477	.81832	.58896	.80816	55
6	.54610	.83772	.56064	.82806	.57501	.81815	.58920	.80799	54
7	.54635	.83756	.56088	.82790	.57524	.81798	.58943	.80782	53
8	.54659	.83740	.56112	.82773	.57548	.81782	.58967	.80765	52
9	.54683	.83724	.56136	.82757	.57572	.81765	.58990	.80748	51
10	.54708	.83708	.56160	.82741	.57596	.81748	.59014	.80730	50
11	.54732	.83692	.56184	.82724	.57619	.81731	.59037	.80713	49
12	.54756	.83676	.56208	.82708	.57643	.81714	.59061	.80696	48
13	.54781	.83660	.56232	.82692	.57667	.81698	.59084	.80679	47
14	.54805	.83645	.56256	.82675	.57691	.81681	.59108	.80662	46
15	.54829	.83629	.56280	.82659	.57715	.81664	.59131	.80644	45
16	.54854	.83613	.56305	.82643	.57738	.81647	.59154	.80627	44
17	.54878	.83597	.56329	.82626	.57762	.81631	.59178	.80610	43
18	.54902	.83581	.56353	.82610	.57786	.81614	.59201	.80593	42
19	.54927	.83565	.56377	.82593	.57810	.81597	.59225	.80576	41
20	.54951	.83549	.56401	.82577	.57833	.81580	.59248	.80558	40
21	.54975	.83533	.56425	.82561	.57857	.81563	.59272	.80541	39
22	.54999	.83517	.56449	.82544	.57881	.81546	.59295	.80524	38
23	.55024	.83501	.56473	.82528	.57904	.81530	.59318	.80507	37
24	.55048	.83485	.56497	.82511	.57928	.81513	.59342	.80489	36
25	.55072	.83469	.56521	.82495	.57952	.81496	.59365	.80472	35
26	.55097	.83453	.56545	.82478	.57976	.81479	.59389	.80455	34
27	.55121	.83437	.56569	.82462	.57999	.81462	.59412	.80438	33
28	.55145	.83421	.56593	.82446	.58023	.81445	.59436	.80420	32
29	.55169	.83405	.56617	.82429	.58047	.81428	.59459	.80403	31
30	.55194	.83389	.56641	.82413	.58070	.81412	.59482	.80386	30
31	.55218	.83373	.56665	.82396	.58094	.81395	.59506	.80368	29
32	.55242	.83356	.56689	.82380	.58118	.81378	.59529	.80351	28
33	.55266	.83340	.56713	.82363	.58141	.81361	.59552	.80334	27
34	.55291	.83324	.56736	.82347	.58165	.81344	.59576	.80316	26
35	.55315	.83308	.56760	.82330	.58189	.81327	.59599	.80299	25
36	.55339	.83292	.56784	.82314	.58212	.81310	.59622	.80282	24
37	.55363	.83276	.56808	.82297	.58236	.81293	.59646	.80264	23
38	.55388	.83260	.56832	.82281	.58260	.81276	.59669	.80247	22
39	.55412	.83244	.56856	.82264	.58283	.81259	.59693	.80230	21
40	.55436	.83228	.56880	.82248	.58307	.81242	.59716	.80212	20
41	.55460	.83212	.56904	.82231	.58330	.81225	.59739	.80195	19
42	.55484	.83195	.56928	.82214	.58354	.81208	.59763	.80178	18
43	.55509	.83179	.56952	.82198	.58378	.81191	.59786	.80160	17
44	.55533	.83163	.56976	.82181	.58401	.81174	.59809	.80143	16
45	.55557	.83147	.57000	.82165	.58425	.81157	.59832	.80125	15
46	.55581	.83131	.57024	.82148	.58449	.81140	.59856	.80108	14
47	.55605	.83115	.57047	.82132	.58472	.81123	.59879	.80091	13
48	.55630	.83098	.57071	.82115	.58496	.81106	.59902	.80073	12
49	.55654	.83082	.57095	.82098	.58519	.81089	.59926	.80056	11
50	.55678	83066	.57119	.82082	.58543	.81072	.59949	.80038	10
51	.55702	.83050	.57143	.82065	.58567	.81055	.59972	.80021	9
52	.55726	.83034	.57167	.82048	.58590	.81038	.59995	.80003	8
53	.55750	.83017	.57191	.82032	.58614	.81021	.60019	.79986	7
54	.55775	.83001	.57215	.82015	.58637	.81004	.60042	.79968	6
55	.55799	.82985	.57238	.81999	.58661	.80987	.60065	.79951	5
56	.55823	.82969	.57262	.81982	.58684	.80970	.60089	.79934	4
57	.55847	.82953	.57286	.81965	.58708	.80953	.60112	.79916	3
58	.55871	.82936	.57310	.81949	.58731	.80936	.60135	.79899	2
59	.55895	.82920	.57334	.81932	.58755	.80919	.60158	.79881	1
60	.55919	.82904	.57358	.81915	.58779	.80902	.60182	.79864	0
′	56° Cosine	56° Sine	55° Cosine	55° Sine	54° Cosine	54° Sine	53° Cosine	53° Sine	′

′	37° Sine	37° Cosine	38° Sine	38° Cosine	39° Sine	39° Cosine	40° Sine	40° Cosine	′
0	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	60
1	.60205	.79846	.61589	.78783	.62955	.77696	.64301	.76586	59
2	.60228	.79829	.61612	.78765	.62977	.77678	.64323	.76567	58
3	.60251	.79811	.61635	.78747	.63000	.77660	.64346	.76548	57
4	.60274	.79793	.61658	.78729	.63022	.77641	.64368	.76530	56
5	.60298	.79776	.61681	.78711	.63045	.77623	.64390	.76511	55
6	.60321	.79758	.61704	.78694	.63068	.77605	.64412	.76492	54
7	.60344	.79741	.61726	.78676	.63090	.77586	.64435	.76473	53
8	.60367	.79723	.61749	.78658	.63113	.77568	.64457	.76455	52
9	.60390	.79706	.61772	.78640	.63135	.77550	.64479	.76436	51
10	.60414	.79688	.61795	.78622	.63158	.77531	.64501	.76417	50
11	.60437	.79671	.61818	.78604	.63180	.77513	.64524	.76398	49
12	.60460	.79653	.61841	.78586	.63203	.77494	.64546	.76380	48
13	.60483	.79635	.61864	.78568	.63225	.77476	.64568	.76361	47
14	.60506	.79618	.61887	.78550	.63248	.77458	.64590	.76342	46
15	.60529	.79600	.61909	.78532	.63271	.77439	.64612	.76323	45
16	.60553	.79583	.61932	.78514	.63293	.77421	.64635	.76304	44
17	.60576	.79565	.61955	.78496	.63316	.77402	.64657	.76286	43
18	.60599	.79547	.61978	.78478	.63338	.77384	.64679	.76267	42
19	.60622	.79530	.62001	.78460	.63361	.77366	.64701	.76248	41
20	.60645	.79512	.62024	.78442	.63383	.77347	.64723	.76229	40
21	.60668	.79494	.62046	.78424	.63406	.77329	.64746	.76210	39
22	.60691	.79477	.62069	.78405	.63428	.77310	.64768	.76192	38
23	.60714	.79459	.62092	.78387	.63451	.77292	.64790	.76173	37
24	.60738	.79441	.62115	.78369	.63473	.77273	.64812	.76154	36
25	.60761	.79424	.62138	.78351	.63496	.77255	.64834	.76135	35
26	.60784	.79406	.62160	.78333	.63518	.77236	.64856	.76116	34
27	.60807	.79388	.62183	.78315	.63540	.77218	.64878	.76097	33
28	.60830	.79371	.62206	.78297	.63563	.77199	.64901	.76078	32
29	.60853	.79353	.62229	.78279	.63585	.77181	.64923	.76059	31
30	.60876	.79335	.62251	.78261	.63608	.77162	.64945	.76041	30
31	.60899	.79318	.62274	.78243	.63630	.77144	.64967	.76022	29
32	.60922	.79300	.62297	.78225	.63653	.77125	.64989	.76003	28
33	.60945	.79282	.62320	.78206	.63675	.77107	.65011	.75984	27
34	.60968	.79264	.62342	.78188	.63698	.77088	.65033	.75965	26
35	.60991	.79247	.62365	.78170	.63720	.77070	.65055	.75946	25
36	.61015	.79229	.62388	.78152	.63742	.77051	.65077	.75927	24
37	.61038	.79211	.62411	.78134	.63765	.77033	.65100	.75908	23
38	.61061	.79193	.62433	.78116	.63787	.77014	.65122	.75889	22
39	.61084	.79176	.62456	.78098	.63810	.76996	.65144	.75870	21
40	.61107	.79158	.62479	.78079	.63832	.76977	.65166	.75851	20
41	.61130	.79140	.62502	.78061	.63854	.76959	.65188	.75832	19
42	.61153	.79122	.62524	.78043	.63877	.76940	.65210	.75813	18
43	.61176	.79105	.62547	.78025	.63899	.76921	.65232	.75794	17
44	.61199	.79087	.62570	.78007	.63922	.76903	.65254	.75775	16
45	.61222	.79069	.62592	.77988	.63944	.76884	.65276	.75756	15
46	.61245	.79051	.62615	.77970	.63966	.76866	.65298	.75738	14
47	.61268	.79033	.62638	.77952	.63989	.76847	.65320	.75719	13
48	.61291	.79016	.62660	.77934	.64011	.76828	.65342	.75700	12
49	.61314	.78998	.62683	.77916	.64033	.76810	.65364	.75680	11
50	.61337	.78980	.62706	.77897	.64056	.76791	.65386	.75661	10
51	.61360	.78962	.62728	.77879	.64078	.76772	.65408	.75642	9
52	.61383	.78944	.62751	.77861	.64100	.76754	.65430	.75623	8
53	.61406	.78926	.62774	.77843	.64123	.76735	.65452	.75604	7
54	.61429	.78908	.62796	.77824	.64145	.76717	.65474	.75585	6
55	.61451	.78891	.62819	.77806	.64167	.76698	.65496	.75566	5
56	.61474	.78873	.62842	.77788	.64190	.76679	.65518	.75547	4
57	.61497	.78855	.62864	.77769	.64212	.76661	.65540	.75528	3
58	.61520	.78837	.62887	.77751	.64234	.76642	.65562	.75509	2
59	.61543	.78819	.62909	.77733	.64256	.76623	.65584	.75490	1
60	.61566	.78801	.62932	.77715	.64279	.76604	.65606	.75471	0
′	52° Cosine	52° Sine	51° Cosine	51° Sine	50° Cosine	50° Sine	49° Cosine	49° Sine	′

′	41° Sine	41° Cosine	42° Sine	42° Cosine	43° Sine	43° Cosine	44° Sine	44° Cosine	′
0	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.66891	.74334	.68179	.73155	.69445	.71054	.70690	.70731	1
60	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	48° Cosine	48° Sine	47° Cosine	47° Sine	46° Cosine	46° Sine	45° Cosine	45° Sine	′

′	0° Sec.	0° Co-sec.	1° Sec.	1° Co-sec.	2° Sec.	2° Co-sec.	3° Sec.	3° Co-sec.	′
0	1	Infinite.	1.0001	57.299	1.0006	28.654	1.0014	19.107	60
1	1	3437.70	1.0001	56.359	1.0006	28.417	1.0014	19.002	59
2	1	1718.90	1.0002	55.450	1.0006	28.184	1.0014	18.897	58
3	1	1145.90	1.0002	54.570	1.0006	27.955	1.0014	18.794	57
4	1	859.44	1.0002	53.718	1.0006	27.730	1.0014	18.692	56
5	1	687.55	1.0002	52.891	1.0007	27.508	1.0014	18.591	55
6	1	572.96	1.0002	52.090	1.0007	27.290	1.0015	18.491	54
7	1	491.11	1.0002	51.313	1.0007	27.075	1.0015	18.393	53
8	1	429.72	1.0002	50.558	1.0007	26.864	1.0015	18.295	52
9	1	381.97	1.0002	49.826	1.0007	26.655	1.0015	18.198	51
10	1	343.77	1.0002	49.114	1.0007	26.450	1.0015	18.103	50
11	1	312.52	1.0002	48.422	1.0007	26.249	1.0015	18.008	49
12	1	286.48	1.0002	47.750	1.0007	26.050	1.0016	17.914	48
13	1	264.44	1.0002	47.096	1.0007	25.854	1.0016	17.821	47
14	1	245.55	1.0002	46.460	1.0008	25.661	1.0016	17.730	46
15	1	229.18	1.0002	45.840	1.0008	25.471	1.0016	17.639	45
16	1	214.86	1.0002	45.237	1.0008	25.284	1.0016	17.549	44
17	1	202.22	1.0002	44.650	1.0008	25.100	1.0016	17.460	43
18	1	190.99	1.0002	44.077	1.0008	24.918	1.0017	17.372	42
19	1	180.73	1.0003	43.520	1.0008	24.739	1.0017	17.285	41
20	1	171.89	1.0003	42.976	1.0008	24.562	1.0017	17.198	40
21	1	163.70	1.0003	42.445	1.0008	24.358	1.0017	17.113	39
22	1	156.26	1.0003	41.928	1.0008	24.216	1.0017	17.028	38
23	1	149.47	1.0003	41.423	1.0009	24.047	1.0017	16.944	37
24	1	143.24	1.0003	40.930	1.0009	23.880	1.0018	16.861	36
25	1	137.51	1.0003	40.448	1.0009	23.716	1.0018	16.779	35
26	1	132.22	1.0003	39.978	1.0009	23.553	1.0018	16.698	34
27	1	127.32	1.0003	39.518	1.0009	23.393	1.0018	16.617	33
28	1	122.78	1.0003	39.069	1.0009	23.235	1.0018	16.538	32
29	1	118.54	1.0003	38.631	1.0009	23.079	1.0018	16.459	31
30	1	114.59	1.0003	38.201	1.0009	22.925	1.0019	16.380	30
31	1	110.90	1.0003	37.782	1.0010	22.774	1.0019	16.303	29
32	1	107.43	1.0003	37.371	1.0010	22.624	1.0019	16.226	28
33	1	104.17	1.0004	36.969	1.0010	22.476	1.0019	16.150	27
34	1	101.11	1.0004	36.576	1.0010	22.330	1.0019	16.075	26
35	1	98.223	1.0004	36.191	1.0010	22.186	1.0019	16.000	25
36	1	95.495	1.0004	35.814	1.0010	22.044	1.0020	15.926	24
37	1	92.914	1.0004	35.445	1.0010	21.904	1.0020	15.853	23
38	1.0001	92.469	1.0004	35.084	1.0010	21.765	1.0020	15.780	22
39	1.0001	88.149	1.0004	34.729	1.0011	21.629	1.0020	15.708	21
40	1.0001	85.946	1.0004	34.382	1.0011	21.494	1.0020	15.637	20
41	1.0001	83.849	1.0004	34.042	1.0011	21.360	1.0021	15.566	19
42	1.0001	81.853	1.0004	33.708	1.0011	21.228	1.0021	15.496	18
43	1.0001	79.950	1.0004	33.381	1.0011	21.098	1.0021	15.427	17
44	1.0001	78.133	1.0004	33.060	1.0011	20.970	1.0021	15.358	16
45	1.0001	76.396	1.0005	32.745	1.0011	20.843	1.0021	15.290	15
46	1.0001	74.736	1.0005	32.437	1.0012	20.717	1.0022	15.222	14
47	1.0001	73.146	1.0005	32.134	1.0012	20.593	1.0022	15.155	13
48	1.0001	71.622	1.0005	31.836	1.0012	20.471	1.0022	15.089	12
49	1.0001	71.160	1.0005	31.544	1.0012	20.350	1.0022	15.023	11
50	1.0001	68.757	1.0005	31.257	1.0012	20.230	1.0022	14.958	10
51	1.0001	67.409	1.0005	30.976	1.0012	20.112	1.0023	14.893	9
52	1.0001	66.113	1.0005	30.699	1.0012	19.995	1.0023	14.829	8
53	1.0001	64.866	1.0005	30.428	1.0013	19.880	1.0023	14.765	7
54	1.0001	63.664	1.0005	30.161	1.0013	19.766	1.0023	14.702	6
55	1.0001	62.507	1.0005	29.899	1.0013	19.653	1.0023	14.640	5
56	1.0001	61.391	1.0006	29.641	1.0013	19.541	1.0024	14.578	4
57	1.0001	61.314	1.0006	29.388	1.0013	19.431	1.0024	14.517	3
58	1.0001	59.274	1.0006	29.139	1.0013	19.322	1.0024	14.456	2
59	1.0001	58.270	1.0006	28.894	1.0013	19.214	1.0024	14.395	1
60	1.0001	57.299	1.0006	28.654	1.0014	19.107	1.0024	14.335	0
′	89° Co-sec.	89° Sec.	88° Co-sec.	88° Sec.	87° Co-sec.	87° Sec.	86° Co-sec.	86° Sec.	′

′	4° Sec.	4° Co-sec.	5° Sec.	5° Co-sec.	6° Sec.	6° Co-sec.	7° Sec.	7° Co-sec.	′
0	1.0024	14.335	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	60
1	1.0025	14.276	1.0038	11.436	1.0055	9.5404	1.0075	8.1861	59
2	1.0025	14.217	1.0039	11.398	1.0056	9.5141	1.0076	8.1668	58
3	1.0025	14.159	1.0039	11.360	1.0056	9.4880	1.0076	8.1476	57
4	1.0025	14.101	1.0039	11.323	1.0056	9.4620	1.0076	8.1285	56
5	1.0025	14.043	1.0039	11.286	1.0057	9.4362	1.0077	8.1094	55
6	1.0026	13.986	1.0040	11.249	1.0057	9.4105	1.0077	8.0905	54
7	1.0026	13.930	1.0040	11.213	1.0057	9.3850	1.0078	8.0717	53
8	1.0026	13.874	1.0040	11.176	1.0057	9.3596	1.0078	8.0529	52
9	1.0026	13.818	1.0040	11.140	1.0058	9.3343	1.0078	8.0342	51
10	1.0026	13.763	1.0041	11.104	1.0058	9.3092	1.0079	8.0156	50
11	1.0027	13.708	1.0041	11.069	1.0058	9.2842	1.0079	7.9971	49
12	1.0027	13.654	1.0041	11.033	1.0059	9.2593	1.0079	7.9787	48
13	1.0027	13.600	1.0041	10.988	1.0059	9.2346	1.0080	7.9604	47
14	1.0027	13.547	1.0042	10.963	1.0059	9.2100	1.0080	7.9421	46
15	1.0027	13.494	1.0042	10.929	1.0060	9.1855	1.0080	7.9240	45
16	1.0028	13.441	1.0042	10.894	1.0060	9.1612	1.0081	7.9059	44
17	1.0028	13.389	1.0043	10.860	1.0060	9.1370	1.0081	7.8879	43
18	1.0028	13.337	1.0043	10.826	1.0061	9.1129	1.0082	7.8700	42
19	1.0028	13.286	1.0043	10.792	1.0061	9.0890	1.0082	7.8522	41
20	1.0029	13.235	1.0043	10.758	1.0061	9.0651	1.0082	7.8344	40
21	1.0029	13.184	1.0044	10.725	1.0062	9.0414	1.0083	7.8168	39
22	1.0029	13.134	1.0044	10.692	1.0062	9.0179	1.0083	7.7992	38
23	1.0029	13.084	1.0044	10.659	1.0062	8.9944	1.0084	7.7817	37
24	1.0029	13.034	1.0044	10.626	1.0063	8.9711	1.0084	7.7642	36
25	1.0030	12.985	1.0045	10.593	1.0063	8.9479	1.0084	7.7469	35
26	1.0030	12.937	1.0045	10.561	1.0063	8.9248	1.0085	7.7296	34
27	1.0030	12.888	1.0045	10.529	1.0064	8.9018	1.0085	7.7124	33
28	1.0030	12.840	1.0046	10.497	1.0064	8.8790	1.0085	7.6953	32
29	1.0031	12.793	1.0046	10.465	1.0064	8.8563	1.0086	7.6783	31
30	1.0031	12.745	1.0046	10.433	1.0065	8.8337	1.0086	7.6613	30
31	1.0031	12.698	1.0046	10.402	1.0065	8.8112	1.0087	7.6444	29
32	1.0031	12.652	1.0047	10.371	1.0065	8.7888	1.0087	7.6276	28
33	1.0032	12.606	1.0047	10.340	1.0066	8.7665	1.0087	7.6108	27
34	1.0032	12.560	1.0047	10.309	1.0066	8.7444	1.0088	7.5942	26
35	1.0032	12.514	1.0048	10.278	1.0066	8.7223	1.0088	7.5776	25
36	1.0032	12.469	1.0048	10.248	1.0067	8.7004	1.0089	7.5611	24
37	1.0032	12.424	1.0048	10.217	1.0067	8.6786	1.0089	7.5446	23
38	1.0033	12.379	1.0048	10.187	1.0067	8.6569	1.0089	7.5282	22
39	1.0033	12.335	1.0049	10.157	1.0068	8.6353	1.0090	7.5119	21
40	1.0033	12.291	1.0049	10.127	1.0068	8.6138	1.0090	7.4957	20
41	1.0033	12.248	1.0049	10.098	1.0068	8.5924	1.0090	7.4795	19
42	1.0034	12.204	1.0050	10.068	1.0069	8.5711	1.0091	7.4634	18
43	1.0034	12.161	1.0050	10.039	1.0069	8.5499	1.0091	7.4474	17
44	1.0034	12.118	1.0050	10.010	1.0069	8.5289	1.0092	7.4315	16
45	1.0034	12.076	1.0050	9.9812	1.0070	8.5079	1.0092	7.4156	15
46	1.0035	12.034	1.0051	9.9525	1.0070	8.4871	1.0092	7.3998	14
47	1.0035	11.992	1.0051	9.9239	1.0070	8.4663	1.0093	7.3840	13
48	1.0035	11.950	1.0051	9.8955	1.0071	8.4457	1.0093	7.3683	12
49	1.0035	11.909	1.0052	9.8672	1.0071	8.4251	1.0094	7.3527	11
50	1.0036	11.868	1.0052	9.8391	1.0071	8.4046	1.0094	7.3372	10
51	1.0036	11.828	1.0052	9.8112	1.0072	8.3843	1.0094	7.3217	9
52	1.0036	11.787	1.0053	9.7834	1.0072	8.3640	1.0095	7.3063	8
53	1.0036	11.747	1.0053	9.7558	1.0073	8.3439	1.0095	7.2909	7
54	1.0037	11.707	1.0053	9.7283	1.0073	8.3238	1.0096	7.2757	6
55	1.0037	11.668	1.0053	9.7010	1.0073	8.3039	1.0096	7.2604	5
56	1.0037	11.628	1.0054	9.6739	1.0074	8.2840	1.0097	7.2453	4
57	1.0037	11.589	1.0054	9.6469	1.0074	8.2642	1.0097	7.2302	3
58	1.0038	11.550	1.0054	9.6200	1.0074	8.2446	1.0097	7.2152	2
59	1.0038	11.512	1.0055	9.5933	1.0075	8.2250	1.0098	7.2002	1
60	1.0038	11.474	1.0055	9.5668	1.0075	8.2055	1.0098	7.1853	0
′	85° Co-sec.	85° Sec.	84° Co-sec.	84° Sec.	83° Co-sec.	83° Sec.	82° Co-sec.	82° Sec.	′

′	8° Sec.	8° Co-sec.	9° Sec.	9° Co-sec.	10° Sec.	10° Co-sec.	11° Sec.	11° Co-sec.	′
0	1.0098	7.1853	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	60
1	1.0099	7.1704	1.0125	6.3807	1.0155	5.7493	1.0188	5.2330	59
2	1.0099	7.1557	1.0125	6.3690	1.0155	5.7398	1.0188	5.2252	58
3	1.0099	7.1409	1.0126	6.3574	1.0156	5.7304	1.0189	5.2174	57
4	1.0100	7.1263	1.0126	6.3458	1.0156	5.7210	1.0189	5.2097	56
5	1.0100	7.1117	1.0127	6.3343	1.0157	5.7117	1.0190	5.2019	55
6	1.0101	7.0972	1.0127	6.3228	1.0157	5.7023	1.0191	5.1942	54
7	1.0101	7.0827	1.0128	6.3113	1.0158	5.6930	1.0191	5.1865	53
8	1.0102	7.0683	1.0128	6.2999	1.0158	5.6838	1.0192	5.1788	52
9	1.0102	7.0539	1.0129	6.2885	1.0159	5.6745	1.0192	5.1712	51
10	1.0102	7.0396	1.0129	6.2772	1.0159	5.6653	1.0193	5.1636	50
11	1.0103	7.0254	1.0130	6.2659	1.0160	5.6561	1.0193	5.1560	49
12	1.0103	7.0112	1.0130	6.2546	1.0160	5.6470	1.0194	5.1484	48
13	1.0104	6.9971	1.0131	6.2434	1.0161	5.6379	1.0195	5.1409	47
14	1.0104	6.9830	1.0131	6.2322	1.0162	5.6288	1.0195	5.1333	46
15	1.0104	6.9690	1.0132	6.2211	1.0162	5.6197	1.0196	5.1258	45
16	1.0105	6.9550	1.0132	6.2100	1.0163	5.6107	1.0196	5.1183	44
17	1.0105	6.9411	1.0133	6.1990	1.0163	5.6017	1.0197	5.1109	43
18	1.0106	6.9273	1.0133	6.1880	1.0164	5.5928	1.0198	5.1034	42
19	1.0106	6.9135	1.0134	6.1770	1.0164	5.5838	1.0198	5.0960	41
20	1.0107	6.8998	1.0134	6.1661	1.0165	5.5749	1.0199	5.0886	40
21	1.0107	6.8861	1.0135	6.1552	1.0165	5.5660	1.0199	5.0812	39
22	1.0107	6.8725	1.0135	6.1443	1.0166	5.5572	1.0200	5.0739	38
23	1.0108	6.8589	1.0136	6.1335	1.0166	5.5484	1.0201	5.0666	37
24	1.0108	6.8454	1.0136	6.1227	1.0167	5.5396	1.0201	5.0593	36
25	1.0109	6.8320	1.0136	6.1120	1.0167	5.5308	1.0202	5.0520	35
26	1.0109	6.8185	1.0137	6.1013	1.0168	5.5221	1.0202	5.0447	34
27	1.0110	6.8052	1.0137	6.0906	1.0169	5.5134	1.0203	5.0375	33
28	1.0110	6.7919	1.0138	6.0800	1.0169	5.5047	1.0204	5.0302	32
29	1.0111	6.7787	1.0138	6.0694	1.0170	5.4960	1.0204	5.0230	31
30	1.0111	6.7655	1.0139	6.0588	1.0170	5.4874	1.0205	5.0158	30
31	1.0111	6.7523	1.0139	6.0483	1.0171	5.4788	1.0205	5.0087	29
32	1.0112	6.7392	1.0140	6.0379	1.0171	5.4702	1.0206	5.0015	28
33	1.0112	6.7263	1.0140	6.0274	1.0172	5.4617	1.0207	4.9944	27
34	1.0113	6.7132	1.0141	6.0170	1.0172	5.4532	1.0207	4.9873	26
35	1.0113	6.7003	1.0141	6.0066	1.0173	5.4447	1.0208	4.9802	25
36	1.0114	6.6874	1.0142	5.9963	1.0174	5.4362	1.0208	4.9732	24
37	1.0114	6.6745	1.0142	5.9860	1.0174	5.4278	1.0209	4.9661	23
38	1.0115	6.6617	1.0143	5.9758	1.0175	5.4194	1.0210	4.9591	22
39	1.0115	6.6490	1.0143	5.9655	1.0175	5.4110	1.0210	4.9521	21
40	1.0115	6.6363	1.0144	5.9554	1.0176	5.4026	1.0211	4.9452	20
41	1.0116	6.6237	1.0144	5.9452	1.0176	5.3943	1.0211	4.9382	19
42	1.0116	6.6111	1.0145	5.9351	1.0177	5.3860	1.0212	4.9313	18
43	1.0117	6.5985	1.0145	5.9250	1.0177	5.3777	1.0213	4.9243	17
44	1.0117	6.5860	1.0146	5.9150	1.0178	5.3695	1.0213	4.9175	16
45	1.0118	6.5736	1.0146	5.9049	1.0179	5.3612	1.0214	4.9106	15
46	1.0118	6.5612	1.0147	5.8950	1.0179	5.3530	1.0215	4.9037	14
47	1.0119	6.5488	1.0147	5.8850	1.0180	5.3449	1.0215	4.8969	13
48	1.0119	6.5365	1.0148	5.8751	1.0180	5.3367	1.0216	4.8901	12
49	1.0119	6.5243	1.0148	5.8652	1.0181	5.3286	1.0216	4.8833	11
50	1.0120	6.5121	1.0149	5.8554	1.0181	5.3205	1.0217	4.8765	10
51	1.0120	6.4999	1.0150	5.8456	1.0182	5.3124	1.0218	4.8697	9
52	1.0121	6.4878	1.0150	5.8358	1.0182	5.3044	1.0218	4.8630	8
53	1.0121	6.4757	1.0151	5.8261	1.0183	5.2963	1.0219	4.8563	7
54	1.0122	6.4637	1.0151	5.8163	1.0184	5.2883	1.0220	4.8496	6
55	1.0122	6.4517	1.0152	5.8067	1.0184	5.2803	1.0220	4.8429	5
56	1.0123	6.4398	1.0152	5.7970	1.0185	5.2724	1.0221	4.8362	4
57	1.0123	6.4279	1.0153	5.7874	1.0185	5.2645	1.0221	4.8296	3
58	1.0124	6.4160	1.0153	5.7778	1.0186	5.2566	1.0222	4.8229	2
59	1.0124	6.4042	1.0154	5.7683	1.0186	5.2487	1.0223	4.8163	1
60	1.0125	6.3924	1.0154	5.7588	1.0187	5.2408	1.0223	4.8097	0
′	Co-sec. 81°	Sec. 81°	Co-sec. 80°	Sec. 80°	Co-sec. 79°	Sec. 79°	Co-sec. 78°	Sec. 78°	′

′	12°		13°		14°		15°		′
	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	
0	1.0223	4.8097	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	60
1	1.0224	4.8032	1.0264	4.4398	1.0307	4.1287	1.0353	3.8595	59
2	1.0225	4.7966	1.0264	4.4342	1.0308	4.1239	1.0354	3.8553	58
3	1.0225	4.7901	1.0265	4.4287	1.0308	4.1191	1.0355	3.8512	57
4	1.0226	4.7835	1.0266	4.4231	1.0309	4.1144	1.0356	3.8470	56
5	1.0226	4.7770	1.0266	4.4176	1.0310	4.1096	1.0357	3.8428	55
6	1.0227	4.7706	1.0267	4.4121	1.0311	4.1048	1.0358	3.8387	54
7	1.0228	4.7641	1.0268	4.4065	1.0311	4.1001	1.0358	3.8346	53
8	1.0228	4.7576	1.0268	4.4011	1.0312	4.0953	1.0359	3.8304	52
9	1.0229	4.7512	1.0269	4.3956	1.0313	4.0906	1.0360	3.8263	51
10	1.0230	4.7448	1.0270	4.3910	1.0314	4.0859	1.0361	3.8222	50
11	1.0230	4.7384	1.0271	4.3847	1.0314	4.0812	1.0362	3.8181	49
12	1.0231	4.7320	1.0271	4.3792	1.0315	4.0765	1.0362	3.8140	48
13	1.0232	4.7257	1.0272	4.3738	1.0316	4.0718	1.0363	3.8100	47
14	1.0232	4.7193	1.0273	4.3684	1.0317	4.0672	1.0364	3.8059	46
15	1.0233	4.7130	1.0273	4.3630	1.0317	4.0625	1.0365	3.8018	45
16	1.0234	4.7067	1.0274	4.3576	1.0318	4.0579	1.0366	3.7978	44
17	1.0234	4.7004	1.0275	4.3522	1.0319	4.0532	1.0367	3.7937	43
18	1.0235	4.6942	1.0276	4.3469	1.0320	4.0486	1.0367	3.7897	42
19	1.0235	4.6879	1.0276	4.3415	1.0320	4.0440	1.0368	3.7857	41
20	1.0236	4.6817	1.0277	4.3362	1.0321	4.0394	1.0369	3.7816	40
21	1.0237	4.6754	1.0278	4.3309	1.0322	4.0348	1.0370	3.7776	39
22	1.0237	4.6692	1.0278	4.3256	1.0323	4.0302	1.0371	3.7736	38
23	1.0238	4.6631	1.0279	4.3203	1.0323	4.0256	1.0371	3.7697	37
24	1.0239	4.6569	1.0280	4.3150	1.0324	4.0211	1.0372	3.7657	36
25	1.0239	4.6507	1.0280	4.3098	1.0325	4.0165	1.0373	3.7617	35
26	1.0240	4.6446	1.0281	4.3045	1.0326	4.0120	1.0374	3.7577	34
27	1.0241	4.6385	1.0282	4.2993	1.0327	4.0074	1.0375	3.7538	33
28	1.0241	4.6324	1.0283	4.2941	1.0327	4.0029	1.0376	3.7498	32
29	1.0242	4.6263	1.0283	4.2888	1.0328	3.9984	1.0376	3.7459	31
30	1.0243	4.6202	1.0284	4.2836	1.0329	3.9939	1.0377	3.7420	30
31	1.0243	4.6142	1.0285	4.2785	1.0330	3.9894	1.0378	3.7380	29
32	1.0244	4.6081	1.0285	4.2733	1.0330	3.9850	1.0379	3.7341	28
33	1.0245	4.6021	1.0286	4.2681	1.0331	3.9805	1.0380	3.7302	27
34	1.0245	4.5961	1.0287	4.2630	1.0332	3.9760	1.0381	3.7263	26
35	1.0246	4.5901	1.0288	4.2579	1.0333	3.9716	1.0382	3.7224	25
36	1.0247	4.5841	1.0288	4.2527	1.0334	3.9672	1.0382	3.7186	24
37	1.0247	4.5782	1.0289	4.2476	1.0334	3.9627	1.0383	3.7147	23
38	1.0248	4.5722	1.0290	4.2425	1.0335	3.9583	1.0384	3.7108	22
39	1.0249	4.5663	1.0291	4.2375	1.0336	3.9539	1.0385	3.7070	21
40	1.0249	4.5604	1.0291	4.2324	1.0337	3.9495	1.0386	3.7031	20
41	1.0250	4.5545	1.0292	4.2273	1.0338	3.9451	1.0387	3.6993	19
42	1.0251	4.5486	1.0293	4.2223	1.0338	3.9408	1.0387	3.6955	18
43	1.0251	4.5428	1.0293	4.2173	1.0339	3.9364	1.0388	3.6917	17
44	1.0252	4.5369	1.0294	4.2122	1.0340	3.9320	1.0389	3.6878	16
45	1.0253	4.5311	1.0295	4.2072	1.0341	3.9277	1.0390	3.6840	15
46	1.0253	4.5253	1.0296	4.2022	1.0341	3.9234	1.0391	3.6802	14
47	1.0254	4.5195	1.0296	4.1972	1.0342	3.9199	1.0392	3.6765	13
48	1.0255	4.5137	1.0297	4.1923	1.0343	3.9147	1.0393	3.6727	12
49	1.0255	4.5079	1.0298	4.1873	1.0344	3.9104	1.0393	3.6689	11
50	1.0256	4.5021	1.0299	4.1824	1.0345	3.9061	1.0394	3.6651	10
51	1.0257	4.4964	1.0299	4.1774	1.0345	3.9018	1.0395	3.6614	9
52	1.0257	4.4907	1.0300	4.1725	1.0346	3.8976	1.0396	3.6576	8
53	1.0258	4.4850	1.0301	4.1676	1.0347	3.8933	1.0397	3.6539	7
54	1.0259	4.4793	1.0302	4.1627	1.0348	3.8890	1.0398	3.6502	6
55	1.0260	4.4736	1.0302	4.1578	1.0349	3.8848	1.0399	3.6464	5
56	1.0260	4.4679	1.0303	4.1529	1.0349	3.8805	1.0399	3.6427	4
57	1.0261	4.4623	1.0304	4.1481	1.0350	3.8763	1.0400	3.6390	3
58	1.0262	4.4566	1.0305	4.1432	1.0351	3.8721	1.0401	3.6353	2
59	1.0262	4.4510	1.0305	4.1384	1.0352	3.8679	1.0402	3.6316	1
60	1.0263	4.4454	1.0306	4.1336	1.0353	3.8637	1.0403	3.6279	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	77°		76°		75°		74°		

′	16° Sec.	16° Co-sec.	17° Sec.	17° Co-sec.	18° Sec.	18° Co-sec.	19° Sec.	19° Co-sec.	′
0	1.0403	3.6279	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	60
1	1.0404	3.6243	1.0458	3.4170	1.0516	3.2332	1.0577	3.0690	59
2	1.0405	3.6206	1.0459	3.4138	1.0517	3.2303	1.0578	3.0664	58
3	1.0406	3.6169	1.0460	3.4106	1.0518	3.2274	1.0579	3.0638	57
4	1.0406	3.6133	1.0461	3.4073	1.0519	3.2245	1.0580	3.0612	56
5	1.0407	5.6096	1.0461	3.4041	1.0520	3.2216	1.0581	3.0586	55
6	1.0408	3.6060	1.0462	3.4009	1.0521	3.2188	1.0582	3.0561	54
7	1.0409	3.6024	1.0463	3.3977	1.0522	3.2159	1.0584	3.0535	53
8	1.0410	3.5987	1.0464	3.3945	1.0523	3.2131	1.0585	3.0509	52
9	1.0411	3.5951	1.0465	3.3913	1.0524	3.2102	1.0586	3.0484	51
10	1.0412	3.5915	1.0466	3.3881	1.0525	3.2074	1.0587	3.0458	50
11	1.0413	3.5879	1.0467	3.3849	1.0526	3.2045	1.0588	3.0433	49
12	1.0413	3.5843	1.0468	3.3817	1.0527	3.2017	1.0589	3.0407	48
13	1.0414	3.5807	1.0469	3.3785	1.0528	3.1989	1.0590	3.0382	47
14	1.0415	3.5772	1.0470	3.3754	1.0529	3.1960	1.0591	3.0357	46
15	1.0416	3.5736	1.0471	3.3722	1.0530	3.1932	1.0592	3.0331	45
16	1.0417	3.5700	1.0472	3.3690	1.0531	3.1904	1.0593	3.0306	44
17	1.0418	3.5665	1.0473	3.3659	1.0532	3.1876	1.0594	3.0281	43
18	1.0419	3.5629	1.0474	3.3627	1.0533	3.1848	1.0595	3.0256	42
19	1.0420	3.5594	1.0475	3.3596	1.0534	3.1820	1.0596	3.0231	41
20	1.0420	3.5559	1.0476	3.3565	1.0535	3.1792	1.0598	3.0206	40
21	1.0421	3.5523	1.0477	3.3534	1.0536	3.1764	1.0599	3.0181	39
22	1.0422	3.5488	1.0478	3.3502	1.0537	3.1736	1.0600	3.0156	38
23	1.0423	3.5453	1.0478	3.3471	1.0538	3.1708	1.0601	3.0131	37
24	1.0424	3.5418	1.0479	3.3440	1.0539	3.1681	1.0602	3.0106	36
25	1.0425	3.5383	1.0480	3.3409	1.0540	3.1653	1.0603	3.0081	35
26	1.0426	3.5348	1.0481	3.3378	1.0541	3.1625	1.0604	3.0056	34
27	1.0427	3.5313	1.0482	3.3347	1.0542	3.1598	1.0605	3.0031	33
28	1.0428	3.5279	1.0483	3.3316	1.0543	3.1570	1.0606	3.0007	32
29	1.0428	3.5244	1.0484	3.3286	1.0544	3.1543	1.0607	2.9982	31
30	1.0429	3.5209	1.0485	3.3255	1.0545	3.1515	1.0608	2.9957	30
31	1.0430	3.5175	1.0486	3.3224	1.0546	3.1488	1.0609	2.9933	29
32	1.0431	3.5140	1.0487	3.3194	1.0547	3.1461	1.0611	2.9908	28
33	1.0432	3.5106	1.0488	3.3163	1.0548	3.1433	1.0612	2.9884	27
34	1.0433	3.5072	1.0489	3.3133	1.0549	3.1406	1.0613	2.9859	26
35	1.0434	3.5037	1.0490	3.3102	1.0550	3.1379	1.0614	2.9835	25
36	1.0435	3.5003	1.0491	3.3072	1.0551	3.1352	1.0615	2.9810	24
37	1.0436	3.4969	1.0492	3.3042	1.0552	3.1325	1.0616	2.9786	23
38	1.0437	3.4935	1.0493	3.3011	1.0553	3.1298	1.0617	2.9762	22
39	1.0438	3.4901	1.0494	3.2981	1.0554	3.1271	1.0618	2.9738	21
40	1.0438	3.4867	1.0495	3.2951	1.0555	3.1244	1.0619	2.9713	20
41	1.0439	3.4833	1.0496	3.2921	1.0556	3.1217	1.0620	2.9689	19
42	1.0440	3.4799	1.0497	3.2891	1.0557	3.1190	1.0622	2.9665	18
43	1.0441	3.4766	1.0498	3.2861	1.0558	3.1163	1.0623	2.9641	17
44	1.0442	3.4732	1.0499	3.2831	1.0559	3.1137	1.0624	2.9617	16
45	1.0443	3.4698	1.0500	3.2801	1.0560	3.1110	1.0625	2.9593	15
46	1.0444	3.4665	1.0501	3.2772	1.0561	3.1083	1.0626	2.9569	14
47	1.0445	3.4632	1.0502	3.2742	1.0562	3.1057	1.0627	2.9545	13
48	1.0446	3.4598	1.0503	3.2712	1.0563	3.1030	1.0628	2.9521	12
49	1.0447	3.4565	1.0504	3.2683	1.0565	3.1004	1.0629	2.9497	11
50	1.0448	3.4532	1.0505	3.2653	1.0566	3.0977	1.0630	2.9474	10
51	1.0448	3.4498	1.0506	3.2624	1.0567	3.0951	1.0632	2.9450	9
52	1.0449	3.4465	1.0507	3.2594	1.0568	3.0925	1.0633	2.9426	8
53	1.0450	3.4432	1.0508	3.2565	1.0569	3.0898	1.0634	2.9402	7
54	1.0451	3.4399	1.0509	3.2535	1.0570	3.0872	1.0635	2.9379	6
55	1.0452	3.4366	1.0510	3.2506	1.0571	3.0846	1.0636	2.9355	5
56	1.0453	3.4334	1.0511	3.2477	1.0572	3.0820	1.0637	2.9332	4
57	1.0454	3.4301	1.0512	3.2448	1.0573	3.0793	1.0638	2.9308	3
58	1.0455	3.4268	1.0513	3.2419	1.0574	3.0767	1.0639	2.9285	2
59	1.0456	3.4236	1.0514	3.2390	1.0575	3.0741	1.0641	2.9261	1
60	1.0457	3.4203	1.0515	3.2361	1.0576	3.0715	1.0642	2.9238	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	73°		72°		71°		70°		

′	20° Sec.	20° Co-sec.	21° Sec.	21° Co-sec.	22° Sec.	22° Co-sec.	23° Sec.	23° Co-sec.	′
0	1.0642	2.9238	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	60
1	1.0643	2.9215	1.0713	2.7883	1.0787	2.6675	1.0865	2.5575	59
2	1.0644	2.9191	1.0714	2.7862	1.0788	2.6656	1.0866	2.5558	58
3	1.0645	2.9168	1.0715	2.7841	1.0789	2.6637	1.0868	2.5540	57
4	1.0646	2.9145	1.0716	2.7820	1.0790	2.6618	1.0869	2.5523	56
5	1.0647	2.9122	1.0717	2.7799	1.0792	2.6599	1.0870	2.5506	55
6	1.0648	2.9098	1.0719	2.7778	1.0793	2.6580	1.0872	2.5488	54
7	1.0650	2.9075	1.0720	2.7757	1.0794	2.6561	1.0873	2.5471	53
8	1.0651	2.9052	1.0721	2.7736	1.0795	2.6542	1.0874	2.5453	52
9	1.0652	2.9029	1.0722	2.7715	1.0797	2.6523	1.0876	2.5436	51
10	1.0653	2.9006	1.0723	2.7694	1.0798	2.6504	1.0877	2.5419	50
11	1.0654	2.8983	1.0725	2.7674	1.0799	2.6485	1.0878	2.5402	49
12	1.0655	2.8960	1.0726	2.7653	1.0801	2.6466	1.0880	2.5384	48
13	1.0656	2.8937	1.0727	2.7632	1.0802	2.6447	1.0881	2.5367	47
14	1.0658	2.8915	1.0728	2.7611	1.0803	2.6428	1.0882	2.5350	46
15	1.0659	2.8892	1.0729	2.7591	1.0804	2.6410	1.0884	2.5333	45
16	1.0660	2.8869	1.0731	2.7570	1.0806	2.6391	1.0885	2.5316	44
17	1.0661	2.8846	1.0732	2.7550	1.0807	2.6372	1.0886	2.5299	43
18	1.0662	2.8824	1.0733	2.7529	1.0808	2.6353	1.0888	2.5281	42
19	1.0663	2.8801	1.0734	2.7509	1.0810	2.6335	1.0889	2.5264	41
20	1.0664	2.8778	1.0736	2.7488	1.0811	2.6316	1.0891	2.5247	40
21	1.0666	2.8756	1.0737	2.7468	1.0812	2.6297	1.0892	2.5230	39
22	1.0667	2.8733	1.0738	2.7447	1.0813	2.6279	1.0893	2.5213	38
23	1.0668	2.8711	1.0739	2.7427	1.0815	2.6260	1.0895	2.5196	37
24	1.0669	2.8688	1.0740	2.7406	1.0816	2.6242	1.0896	2.5179	36
25	1.0670	2.8666	1.0742	2.7386	1.0817	2.6223	1.0897	2.5163	35
26	1.0671	2.8644	1.0743	2.7366	1.0819	2.6205	1.0899	2.5146	34
27	1.0673	2.8621	1.0744	2.7346	1.0820	2.6186	1.0900	2.5129	33
28	1.0674	2.8599	1.0745	2.7325	1.0821	2.6168	1.0902	2.5112	32
29	1.0675	2.8577	1.0747	2.7305	1.0823	2.6150	1.0903	2.5095	31
30	1.0676	2.8554	1.0748	2.7285	1.0824	2.6131	1.0904	2.5078	30
31	1.0677	2.8532	1.0749	2.7265	1.0825	2.6113	1.0906	2.5062	29
32	1.0678	2.8510	1.0750	2.7245	1.0826	2.6095	1.0907	2.5045	28
33	1.0679	2.8488	1.0751	2.7225	1.0828	2.6076	1.0908	2.5028	27
34	1.0681	2.8466	1.0753	2.7205	1.0829	2.6058	1.0910	2.5011	26
35	1.0682	2.8444	1.0754	2.7185	1.0830	2.6040	1.0911	2.4995	25
36	1.0683	2.8422	1.0755	2.7165	1.0832	2.6022	1.0913	2.4978	24
37	1.0684	2.8400	1.0756	2.7145	1.0833	2.6003	1.0914	2.4961	23
38	1.0685	2.8378	1.0758	2.7125	1.0834	2.5985	1.0915	2.4945	22
39	1.0686	2.8356	1.0759	2.7105	1.0836	2.5967	1.0917	2.4928	21
40	1.0688	2.8334	1.0760	2.7085	1.0837	2.5949	1.0918	2.4912	20
41	1.0689	2.8312	1.0761	2.7065	1.0838	2.5931	1.0920	2.4895	19
42	1.0690	2.8290	1.0763	2.7045	1.0840	2.5913	1.0921	2.4879	18
43	1.0691	2.8269	1.0764	2.7026	1.0841	2.5895	1.0922	2.4862	17
44	1.0692	2.8247	1.0765	2.7006	1.0842	2.5877	1.0924	2.4846	16
45	1.0694	2.8225	1.0766	2.6986	1.0844	2.5859	1.0925	2.4829	15
46	1.0695	2.8204	1.0768	2.6967	1.0845	2.5841	1.0927	2.4813	14
47	1.0696	2.8182	1.0769	2.6947	1.0846	2.5823	1.0928	2.4797	13
48	1.0697	2.8160	1.0770	2.6927	1.0847	2.5805	1.0929	2.4780	12
49	1.0698	2.8139	1.0771	2.6908	1.0849	2.5787	1.0931	2.4764	11
50	1.0699	2.8117	1.0773	2.6888	1.0850	2.5770	1.0932	2.4748	10
51	1.0701	2.8096	1.0774	2.6869	1.0851	2.5752	1.0934	2.4731	9
52	1.0702	2.8074	1.0775	2.6849	1.0853	2.5734	1.0935	2.4715	8
53	1.0703	2.8053	1.0776	2.6830	1.0854	2.5716	1.0936	2.4699	7
54	1.0704	2.8032	1.0778	2.6810	1.0855	2.5699	1.0938	2.4683	6
55	1.0705	2.8010	1.0779	2.6791	1.0857	2.5681	1.0939	2.4666	5
56	1.0707	2.7989	1.0780	2.6772	1.0858	2.5663	1.0941	2.4650	4
57	1.0708	2.7968	1.0781	2.6752	1.0859	2.5646	1.0942	2.4634	3
58	1.0709	2.7947	1.0783	2.6733	1.0861	2.5628	1.0943	2.4618	2
59	1.0710	2.7925	1.0784	2.6714	1.0862	2.5610	1.0945	2.4602	1
60	1.0711	2.7904	1.0785	2.6695	1.0864	2.5593	1.0946	2.4586	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	69°		68°		67°		66°		

′	24° Sec.	24° Co-sec.	25° Sec.	25° Co-sec.	26° Sec.	26° Co-sec.	27° Sec.	27° Co-sec.	′
0	1.0946	2.4586	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	60
1	1.0948	2.4570	1.1035	2.3647	1.1127	2.2798	1.1225	2.2014	59
2	1.0949	2.4554	1.1037	2.3632	1.1129	2.2784	1.1226	2.2002	58
3	1.0951	2.4538	1.1038	2.3618	1.1131	2.2771	1.1228	2.1989	57
4	1.0952	2.4522	1.1040	2.3603	1.1132	2.2757	1.1230	2.1977	56
5	1.0953	2.4506	1.1041	2.3588	1.1134	2.2744	1.1231	2.1964	55
6	1.0955	2.4490	1.1043	2.3574	1.1135	2.2730	1.1233	2.1952	54
7	1.0956	2.4474	1.1044	2.3559	1.1137	2.2717	1.1235	2.1939	53
8	1.0958	2.4458	1.1046	2.3544	1.1139	2.2703	1.1237	2.1927	52
9	1.0959	2.4442	1.1047	2.3530	1.1140	2.2690	1.1238	2.1914	51
10	1.0961	2.4426	1.1049	2.3515	1.1142	2.2676	1.1240	2.1902	50
11	1.0962	2.4411	1.1050	2.3501	1.1143	2.2663	1.1242	2.1889	49
12	1.0963	2.4395	1.1052	2.3486	1.1145	2.2650	1.1243	2.1877	48
13	1.0965	2.4379	1.1053	2.3472	1.1147	2.2636	1.1245	2.1865	47
14	1.0966	2.4363	1.1055	2.3457	1.1148	2.2623	1.1247	2.1852	46
15	1.0968	2.4347	1.1056	2.3443	1.1150	2.2610	1.1248	2.1840	45
16	1.0969	2.4332	1.1058	2.3428	1.1151	2.2596	1.1250	2.1828	44
17	1.0971	2.4316	1.1059	2.3414	1.1153	2.2583	1.1252	2.1815	43
18	1.0972	2.4300	1.1061	2.3399	1.1155	2.2570	1.1253	2.1803	42
19	1.0973	2.4285	1.1062	2.3385	1.1156	2.2556	1.1255	2.1791	41
20	1.0975	2.4269	1.1064	2.3371	1.1158	2.2543	1.1257	2.1778	40
21	1.0976	2.4254	1.1065	2.3356	1.1159	2.2530	1.1258	2.1766	39
22	1.0978	2.4238	1.1067	2.3342	1.1161	2.2517	1.1260	2.1754	38
23	1.0979	2.4222	1.1068	2.3328	1.1163	2.2503	1.1262	2.1742	37
24	1.0981	2.4207	1.1070	2.3313	1.1164	2.2490	1.1264	2.1730	36
25	1.0982	2.4191	1.1072	2.3299	1.1166	2.2477	1.1265	2.1717	35
26	1.0984	2.4176	1.1073	2.3285	1.1167	2.2464	1.1267	2.1705	34
27	1.0985	2.4160	1.1075	2.3271	1.1169	2.2451	1.1269	2.1693	33
28	1.0986	2.4145	1.1076	2.3256	1.1171	2.2438	1.1270	2.1681	32
29	1.0988	2.4130	1.1078	2.3242	1.1172	2.2425	1.1272	2.1669	31
30	1.0989	2.4114	1.1079	2.3228	1.1174	2.2411	1.1274	2.1657	30
31	1.0991	2.4099	1.1081	2.3214	1.1176	2.2398	1.1275	2.1645	29
32	1.0992	2.4083	1.1082	2.3200	1.1177	2.2385	1.1277	2.1633	28
33	1.0994	2.4068	1.1084	2.3186	1.1179	2.2372	1.1279	2.1620	27
34	1.0995	2.4053	1.1085	2.3172	1.1180	2.2359	1.1281	2.1608	26
35	1.0997	2.4037	1.1087	2.3158	1.1182	2.2346	1.1282	2.1596	25
36	1.0998	2.4022	1.1088	2.3143	1.1184	2.2333	1.1284	2.1584	24
37	1.1000	2.4007	1.1090	2.3129	1.1185	2.2320	1.1286	2.1572	23
38	1.1001	2.3992	1.1092	2.3115	1.1187	2.2307	1.1287	2.1560	22
39	1.1003	2.3976	1.1093	2.3101	1.1189	2.2294	1.1289	2.1548	21
40	1.1004	2.3961	1.1095	2.3087	1.1190	2.2282	1.1291	2.1536	20
41	1.1005	2.3946	1.1096	2.3073	1.1192	2.2269	1.1293	2.1525	19
42	1.1007	2.3931	1.1098	2.3059	1.1193	2.2256	1.1294	2.1513	18
43	1.1008	2.3916	1.1099	2.3046	1.1195	2.2243	1.1296	2.1501	17
44	1.1010	2.3901	1.1101	2.3032	1.1197	2.2230	1.1298	2.1489	16
45	1.1011	2.3886	1.1102	2.3018	1.1198	2.2217	1.1299	2.1477	15
46	1.1013	2.3871	1.1104	2.3004	1.1200	2.2204	1.1301	2.1465	14
47	1.1014	2.3856	1.1106	2.2990	1.1202	2.2192	1.1303	2.1453	13
48	1.1016	2.3841	1.1107	2.2976	1.1203	2.2179	1.1305	2.1441	12
49	1.1017	2.3826	1.1109	2.2962	1.1205	2.2166	1.1306	2.1430	11
50	1.1019	2.3811	1.1110	2.2949	1.1207	2.2153	1.1308	2.1418	10
51	1.1020	2.3796	1.1112	2.2935	1.1208	2.2141	1.1310	2.1406	9
52	1.1022	2.3781	1.1113	2.2921	1.1210	2.2128	1.1312	2.1394	8
53	1.1023	2.3766	1.1115	2.2907	1.1212	2.2115	1.1313	2.1382	7
54	1.1025	2.3751	1.1116	2.2894	1.1213	2.2103	1.1315	2.1371	6
55	1.1026	2.3736	1.1118	2.2880	1.1215	2.2090	1.1317	2.1359	5
56	1.1028	2.3721	1.1120	2.2866	1.1217	2.2077	1.1319	2.1347	4
57	1.1029	2.3706	1.1121	2.2853	1.1218	2.2065	1.1320	2.1335	3
58	1.1031	2.3691	1.1123	2.2839	1.1220	2.2052	1.1322	2.1324	2
59	1.1032	2.3677	1.1124	2.2825	1.1222	2.2039	1.1324	2.1312	1
60	1.1034	2.3662	1.1126	2.2812	1.1223	2.2027	1.1326	2.1300	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	65°		64°		63°		62°		

′	28° Sec.	28° Co-sec.	29° Sec.	29° Co-sec.	30° Sec.	30° Co-sec.	31° Sec.	31° Co-sec.	′
0	1.1326	2.1300	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	60
1	1.1327	2.1289	1.1435	2.0616	1.1549	1.9990	1.1668	1.9407	59
2	1.1329	2.1277	1.1437	2.0605	1.1551	1.9980	1.1670	1.9397	58
3	1.1331	2.1266	1.1439	2.0594	1.1553	1.9970	1.1672	1.9388	57
4	1.1333	2.1254	1.1441	2.0583	1.1555	1.9960	1.1674	1.9378	56
5	1.1334	2.1242	1.1443	2.0573	1.1557	1.9950	1.1676	1.9369	55
6	1.1336	2.1231	1.1445	2.0562	1.1559	1.9940	1.1678	1.9360	54
7	1.1338	2.1219	1.1446	2.0551	1.1561	1.9930	1.1681	1.9350	53
8	1.1340	2.1208	1.1448	2.0540	1.1562	1.9920	1.1683	1.9341	52
9	1.1341	2.1196	1.1450	2.0530	1.1564	1.9910	1.1685	1.9332	51
10	1.1343	2.1185	1.1452	2.0519	1.1566	1.9900	1.1687	1.9322	50
11	1.1345	2.1173	1.1454	2.0508	1.1568	1.9890	1.1689	1.9313	49
12	1.1347	2.1162	1.1456	2.0498	1.1570	1.9880	1.1691	1.9304	48
13	1.1349	2.1150	1.1458	2.0487	1.1572	1.9870	1.1693	1.9295	47
14	1.1350	2.1139	1.1459	2.0476	1.1574	1.9860	1.1695	1.9285	46
15	1.1352	2.1127	1.1461	2.0466	1.1576	1.9850	1.1697	1.9276	45
16	1.1354	2.1116	1.1463	2.0455	1.1578	1.9840	1.1699	1.9267	44
17	1.1356	2.1104	1.1465	2.0444	1.1580	1.9830	1.1701	1.9258	43
18	1.1357	2.1093	1.1467	2.0434	1.1582	1.9820	1.1703	1.9248	42
19	1.1359	2.1082	1.1469	2.0423	1.1584	1.9811	1.1705	1.9239	41
20	1.1361	2.1070	1.1471	2.0413	1.1586	1.9801	1.1707	1.9230	40
21	1.1363	2.1059	1.1473	2.0402	1.1588	1.9791	1.1709	1.9221	39
22	1.1365	2.1048	1.1474	2.0392	1.1590	1.9781	1.1712	1.9212	38
23	1.1366	2.1036	1.1476	2.0381	1.1592	1.9771	1.1714	1.9203	37
24	1.1368	2.1025	1.1478	2.0370	1.1594	1.9761	1.1716	1.9193	36
25	1.1370	2.1014	1.1480	2.0360	1.1596	1.9752	1.1718	1.9184	35
26	1.1372	2.1002	1.1482	2.0349	1.1598	1.9742	1.1720	1.9175	34
27	1.1373	2.0991	1.1484	2.0339	1.1600	1.9732	1.1722	1.9166	33
28	1.1375	2.0980	1.1486	2.0329	1.1602	1.9722	1.1724	1.9157	32
29	1.1377	2.0969	1.1488	2.0318	1.1604	1.9713	1.1726	1.9148	31
30	1.1379	2.0957	1.1489	2.0308	1.1606	1.9703	1.1728	1.9139	30
31	1.1381	2.0946	1.1491	2.0297	1.1608	1.9693	1.1730	1.9130	29
32	1.1382	2.0935	1.1493	2.0287	1.1610	1.9683	1.1732	1.9121	28
33	1.1384	2.0924	1.1495	2.0276	1.1612	1.9674	1.1734	1.9112	27
34	1.1386	2.0912	1.1497	2.0266	1.1614	1.9664	1.1737	1.9102	26
35	1.1388	2.0901	1.1499	2.0256	1.1616	1.9654	1.1739	1.9093	25
36	1.1390	2.0890	1.1501	2.0245	1.1618	1.9645	1.1741	1.9084	24
37	1.1391	2.0879	1.1503	2.0235	1.1620	1.9635	1.1743	1.9075	23
38	1.1393	2.0868	1.1505	2.0224	1.1622	1.9625	1.1745	1.9066	22
39	1.1395	2.0857	1.1507	2.0214	1.1624	1.9616	1.1747	1.9057	21
40	1.1397	2.0846	1.1508	2.0204	1.1626	1.9606	1.1749	1.9048	20
41	1.1399	2.0835	1.1510	2.0194	1.1628	1.9596	1.1751	1.9039	19
42	1.1401	2.0824	1.1512	2.0183	1.1630	1.9587	1.1753	1.9030	18
43	1.1402	2.0812	1.1514	2.0173	1.1632	1.9577	1.1756	1.9021	17
44	1.1404	2.0801	1.1516	2.0163	1.1634	1.9568	1.1758	1.9013	16
45	1.1406	2.0790	1.1518	2.0152	1.1636	1.9558	1.1760	1.9004	15
46	1.1408	2.0779	1.1520	2.0142	1.1638	1.9549	1.1762	1.8995	14
47	1.1410	2.0768	1.1522	2.0132	1.1640	1.9539	1.1764	1.8986	13
48	1.1411	2.0757	1.1524	2.0122	1.1642	1.9530	1.1766	1.8977	12
49	1.1413	2.0746	1.1526	2.0111	1.1644	1.9520	1.1768	1.8968	11
50	1.1415	2.0735	1.1528	2.0101	1.1646	1.9510	1.1770	1.8959	10
51	1.1417	2.0725	1.1530	2.0091	1.1648	1.9501	1.1772	1.8950	9
52	1.1419	2.0714	1.1531	2.0081	1.1650	1.9491	1.1775	1.8941	8
53	1.1421	2.0703	1.1533	2.0071	1.1652	1.9482	1.1777	1.8932	7
54	1.1422	2.0692	1.1535	2.0061	1.1654	1.9473	1.1779	1.8924	6
55	1.1424	2.0681	1.1537	2.0050	1.1656	1.9463	1.1781	1.8915	5
56	1.1426	2.0670	1.1539	2.0040	1.1658	1.9454	1.1783	1.8906	4
57	1.1428	2.0659	1.1541	2.0030	1.1660	1.9444	1.1785	1.8897	3
58	1.1430	2.0648	1.1543	2.0020	1.1662	1.9435	1.1787	1.8888	2
59	1.1432	2.0637	1.1545	2.0010	1.1664	1.9425	1.1790	1.8879	1
60	1.1433	2.0627	1.1547	2.0000	1.1666	1.9416	1.1792	1.8871	0
′	Co-sec. 61°	Sec. 61°	Co-sec. 60°	Sec. 60°	Co-sec. 59°	Sec. 59°	Co-sec. 58°	Sec. 58°	′

′	32° Sec.	32° Co-sec.	33° Sec.	33° Co-sec.	34° Sec.	34° Co-sec.	35° Sec.	35° Co-sec.	′
0	1.1792	1.8871	1.1924	1.8361	1.2062	1.7883	1.2208	1.7434	60
1	1.1794	1.8862	1.1926	1.8352	1.2064	1.7875	1.2210	1.7427	59
2	1.1796	1.8853	1.1928	1.8344	1.2067	1.7867	1.2213	1.7420	58
3	1.1798	1.8844	1.1930	1.8336	1.2069	1.7860	1.2215	1.7413	57
4	1.1800	1.8836	1.1933	1.8328	1.2072	1.7852	1.2218	1.7405	56
5	1.1802	1.8827	1.1935	1.8320	1.2074	1.7844	1.2220	1.7398	55
6	1.1805	1.8818	1.1937	1.8311	1.2076	1.7837	1.2223	1.7391	54
7	1.1807	1.8809	1.1939	1.8303	1.2079	1.7829	1.2225	1.7384	53
8	1.1809	1.8801	1.1942	1.8295	1.2081	1.7821	1.2228	1.7377	52
9	1.1811	1.8792	1.1944	1.8287	1.2083	1.7814	1.2230	1.7369	51
10	1.1813	1.8783	1.1946	1.8279	1.2086	1.7806	1.2233	1.7362	50
11	1.1815	1.8785	1.1948	1.8271	1.2088	1.7798	1.2235	1.7355	49
12	1.1818	1.8766	1.1951	1.8263	1.2091	1.7791	1.2238	1.7348	48
13	1.1820	1.8757	1.1953	1.8255	1.2093	1.7783	1.2240	1.7341	47
14	1.1822	1.8749	1.1955	1.8246	1.2095	1.7776	1.2243	1.7334	46
15	1.1824	1.8740	1.1958	1.8238	1.2098	1.7768	1.2245	1.7327	45
16	1.1826	1.8731	1.1960	1.8230	1.2100	1.7760	1.2248	1.7319	44
17	1.1828	1.8723	1.1962	1.8222	1.2103	1.7753	1.2250	1.7312	43
18	1.1831	1.8714	1.1964	1.8214	1.2105	1.7745	1.2253	1.7305	42
19	1.1833	1.8706	1.1967	1.8206	1.2107	1.7738	1.2255	1.7298	41
20	1.1835	1.8697	1.1969	1.8198	1.2110	1.7730	1.2258	1.7291	40
21	1.1837	1.8688	1.1971	1.8190	1.2112	1.7723	1.2260	1.7284	39
22	1.1839	1.8680	1.1974	1.8182	1.2115	1.7715	1.2263	1.7277	38
23	1.1841	1.8671	1.1976	1.8174	1.2117	1.7708	1.2265	1.7270	37
24	1.1844	1.8663	1.1978	1.8166	1.2119	1.7700	1.2268	1.7263	36
25	1.1846	1.8654	1.1980	1.8158	1.2122	1.7693	1.2270	1.7256	35
26	1.1848	1.8646	1.1983	1.8150	1.2124	1.7685	1.2273	1.7249	34
27	1.1850	1.8637	1.1985	1.8142	1.2127	1.7678	1.2276	1.7242	33
28	1.1852	1.8629	1.1987	1.8134	1.2129	1.7670	1.2278	1.7234	32
29	1.1855	1.8620	1.1990	1.8126	1.2132	1.7663	1.2281	1.7227	31
30	1.1857	1.8611	1.1992	1.8118	1.2134	1.7655	1.2283	1.7220	30
31	1.1859	1.8603	1.1994	1.8110	1.2136	1.7648	1.2286	1.7213	29
32	1.1861	1.8595	1.1997	1.8102	1.2139	1.7640	1.2288	1.7206	28
33	1.1863	1.8586	1.1999	1.8094	1.2141	1.7633	1.2291	1.7199	27
34	1.1866	1.8578	1.2001	1.8086	1.2144	1.7625	1.2293	1.7192	26
35	1.1868	1.8569	1.2004	1.8078	1.2146	1.7618	1.2296	1.7185	25
36	1.1870	1.8561	1.2006	1.8070	1.2149	1.7610	1.2298	1.7178	24
37	1.1872	1.8552	1.2008	1.8062	1.2151	1.7603	1.2301	1.7171	23
38	1.1874	1.8544	1.2010	1.8054	1.2153	1.7596	1.2304	1.7164	22
39	1.1877	1.8535	1.2013	1.8047	1.2156	1.7588	1.2306	1.7157	21
40	1.1879	1.8527	1.2015	1.8039	1.2158	1.7581	1.2309	1.7151	20
41	1.1881	1.8519	1.2017	1.8031	1.2161	1.7573	1.2311	1.7144	19
42	1.1883	1.8510	1.2020	1.8023	1.2163	1.7566	1.2314	1.7137	18
43	1.1886	1.8502	1.2022	1.8015	1.2166	1.7559	1.2316	1.7130	17
44	1.1888	1.8493	1.2024	1.8007	1.2168	1.7551	1.2319	1.7123	16
45	1.1890	1.8485	1.2027	1.7999	1.2171	1.7544	1.2322	1.7116	15
46	1.1892	1.8477	1.2029	1.7992	1.2173	1.7537	1.2324	1.7109	14
47	1.1894	1.8468	1.2031	1.7984	1.2175	1.7529	1.2327	1.7102	13
48	1.1897	1.8460	1.2034	1.7976	1.2178	1.7522	1.2329	1.7095	12
49	1.1899	1.8452	1.2036	1.7968	1.2180	1.7514	1.2332	1.7088	11
50	1.1901	1.8443	1.2039	1.7960	1.2183	1.7507	1.2335	1.7081	10
51	1.1903	1.8435	1.2041	1.7953	1.2185	1.7500	1.2337	1.7075	9
52	1.1906	1.8427	1.2043	1.7945	1.2188	1.7493	1.2340	1.7068	8
53	1.1908	1.8418	1.2046	1.7937	1.2190	1.7485	1.2342	1.7061	7
54	1.1910	1.8410	1.2048	1.7929	1.2193	1.7478	1.2345	1.7054	6
55	1.1912	1.8402	1.2050	1.7921	1.2195	1.7471	1.2348	1.7047	5
56	1.1915	1.8394	1.2053	1.7914	1.2198	1.7463	1.2350	1.7040	4
57	1.1917	1.8385	1.2055	1.7906	1.2200	1.7456	1.2353	1.7033	3
58	1.1919	1.8377	1.2057	1.7898	1.2203	1.7449	1.2355	1.7027	2
59	1.1921	1.8369	1.2060	1.7891	1.2205	1.7442	1.2358	1.7020	1
60	1.1922	1.8361	1.2062	1.7883	1.2208	1.7434	1.2361	1.7013	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	57°		56°		55°		54°		

′	36° Sec.	36° Co-sec.	37° Sec.	37° Co-sec.	38° Sec.	38° Co-sec.	39° Sec.	39° Co-sec.	′
0	1.2361	1.7013	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	60
1	1.2363	1.7006	1.2524	1.6610	1.2693	1.6237	1.2871	1.5884	59
2	1.2366	1.6999	1.2527	1.6603	1.2696	1.6231	1.2874	1.5879	58
3	1.2368	1.6993	1.2530	1.6597	1.2699	1.6224	1.2877	1.5873	57
4	1.2371	1.6986	1.2532	1.6591	1.2702	1.6218	1.2880	1.5867	56
5	1.2374	1.6979	1.2535	1.6584	1.2705	1.6212	1.2883	1.5862	55
6	1.2376	1.6972	1.2538	1.6578	1.2707	1.6206	1.2886	1.5856	54
7	1.2379	1.6965	1.2541	1.6572	1.2710	1.6200	1.2889	1.5850	53
8	1.2382	1.6959	1.2543	1.6565	1.2713	1.6194	1.2892	1.5845	52
9	1.2384	1.6952	1.2546	1.6559	1.2716	1.6188	1.2895	1.5839	51
10	1.2387	1.6945	1.2549	1.6552	1.2719	1.6182	1.2898	1.5833	50
11	1.2389	1.6938	1.2552	1.6546	1.2722	1.6176	1.2901	1.5828	49
12	1.2392	1.6932	1.2554	1.6540	1.2725	1.6170	1.2904	1.5822	48
13	1.2395	1.6925	1.2557	1.6533	1.2728	1.6164	1.2907	1.5816	47
14	1.2397	1.6918	1.2560	1.6527	1.2731	1.6159	1.2910	1.5811	46
15	1.2400	1.6912	1.2563	1.6521	1.2734	1.6153	1.2913	1.5805	45
16	1.2403	1.6905	1.2565	1.6514	1.2737	1.6147	1.2916	1.5799	44
17	1.2405	1.6898	1.2568	1.6508	1.2739	1.6141	1.2919	1.5794	43
18	1.2408	1.6891	1.2571	1.6502	1.2742	1.6135	1.2922	1.5788	42
19	1.2411	1.6885	1.2574	1.6496	1.2745	1.6129	1.2926	1.5783	41
20	1.2413	1.6878	1.2577	1.6489	1.2748	1.6123	1.2929	1.5777	40
21	1.2416	1.6871	1.2579	1.6483	1.2751	1.6117	1.2932	1.5771	39
22	1.2419	1.6865	1.2582	1.6477	1.2754	1.6111	1.2935	1.5766	38
23	1.2421	1.6858	1.2585	1.6470	1.2757	1.6105	1.2938	1.5760	37
24	1.2424	1.6851	1.2588	1.6464	1.2760	1.6099	1.2941	1.5755	36
25	1.2427	1.6845	1.2591	1.6458	1.2763	1.6093	1.2944	1.5749	35
26	1.2429	1.6838	1.2593	1.6452	1.2766	1.6087	1.2947	1.5743	34
27	1.2432	1.6831	1.2596	1.6445	1.2769	1.6081	1.2950	1.5738	33
28	1.2435	1.6825	1.2599	1.6439	1.2772	1.6077	1.2953	1.5732	32
29	1.2437	1.6818	1.2602	1.6433	1.2775	1.6070	1.2956	1.5727	31
30	1.2440	1.6812	1.2605	1.6427	1.2778	1.6064	1.2960	1.5721	30
31	1.2443	1.6805	1.2607	1.6420	1.2781	1.6058	1.2963	1.5716	29
32	1.2445	1.6798	1.2610	1.6414	1.2784	1.6052	1.2966	1.5710	28
33	1.2448	1.6792	1.2613	1.6408	1.2787	1.6046	1.2969	1.5705	27
34	1.2451	1.6785	1.2616	1.6402	1.2790	1.6040	1.2972	1.5699	26
35	1.2453	1.6779	1.2619	1.6396	1.2793	1.6034	1.2975	1.5694	25
36	1.2456	1.6772	1.2622	1.6389	1.2795	1.6029	1.2978	1.5688	24
37	1.2459	1.6766	1.2624	1.6383	1.2798	1.6023	1.2981	1.5683	23
38	1.2461	1.6759	1.2627	1.6377	1.2801	1.6017	1.2985	1.5677	22
39	1.2464	1.6752	1.2630	1.6371	1.2804	1.6011	1.2988	1.5672	21
40	1.2467	1.6746	1.2633	1.6365	1.2807	1.6005	1.2991	1.5666	20
41	1.2470	1.6739	1.2636	1.6359	1.2810	1.6000	1.2994	1.5661	19
42	1.2472	1.6733	1.2639	1.6352	1.2813	1.5994	1.2997	1.5655	18
43	1.2475	1.6726	1.2641	1.6346	1.2816	1.5988	1.3000	1.5650	17
44	1.2478	1.6720	1.2644	1.6340	1.2819	1.5982	1.3003	1.5644	16
45	1.2480	1.6713	1.2647	1.6334	1.2822	1.5976	1.3006	1.5639	15
46	1.2483	1.6707	1.2650	1.6328	1.2825	1.5971	1.3010	1.5633	14
47	1.2486	1.6700	1.2653	1.6322	1.2828	1.5965	1.3013	1.5628	13
48	1.2488	1.6694	1.2656	1.6316	1.2831	1.5959	1.3016	1.5622	12
49	1.2490	1.6687	1.2659	1.6309	1.2834	1.5953	1.3019	1.5617	11
50	1.2494	1.6681	1.2661	1.6303	1.2837	1.5947	1.3022	1.5611	10
51	1.2497	1.6674	1.2664	1.6297	1.2840	1.5942	1.3025	1.5606	9
52	1.2499	1.6668	1.2667	1.6291	1.2843	1.5936	1.3029	1.5600	8
53	1.2502	1.6661	1.2670	1.6285	1.2846	1.5930	1.3032	1.5595	7
54	1.2505	1.6655	1.2673	1.6279	1.2849	1.5924	1.3035	1.5590	6
55	1.2508	1.6648	1.2676	1.6273	1.2852	1.5919	1.3038	1.5584	5
56	1.2510	1.6642	1.2679	1.6267	1.2855	1.5913	1.3041	1.5579	4
57	1.2513	1.6636	1.2681	1.6261	1.2858	1.5907	1.3044	1.5573	3
58	1.2516	1.6629	1.2684	1.6255	1.2861	1.5901	1.3048	1.5568	2
59	1.2519	1.6623	1.2687	1.6249	1.2864	1.5896	1.3051	1.5563	1
60	1.2521	1.6616	1.2690	1.6243	1.2867	1.5890	1.3054	1.5557	0
′	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	Co-sec.	Sec.	′
	53°		52°		51°		50°		

′	40° Sec.	40° Co-sec.	41° Sec.	41° Co-sec.	42° Sec.	42° Co-sec.	43° Sec.	43° Co-sec.	′
0	1.3054	1.5557	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	60
1	1.3057	1.5552	1.3253	1.5237	1.3460	1.4940	1.3677	1.4658	59
2	1.3060	1.5546	1.3257	1.5232	1.3463	1.4935	1.3681	1.4654	58
3	1.3064	1.5541	1.3260	1.5227	1.3467	1.4930	1.3684	1.4649	57
4	1.3067	1.5536	1.3263	1.5222	1.3470	1.4925	1.3688	1.4644	56
5	1.3070	1.5530	1.3267	1.5217	1.3474	1.4921	1.3692	1.4640	55
6	1.3073	1.5525	1.3270	1.5212	1.3477	1.4916	1.3695	1.4635	54
7	1.3076	1.5520	1.3274	1.5207	1.3481	1.4911	1.3699	1.4631	53
8	1.3080	1.5514	1.3277	1.5202	1.3485	1.4906	1.3703	1.4626	52
9	1.3083	1.5509	1.3280	1.5197	1.3488	1.4901	1.3707	1.4622	51
10	1.3086	1.5503	1.3284	1.5192	1.3492	1.4897	1.3710	1.4617	50
11	1.3089	1.5498	1.3287	1.5187	1.3495	1.4892	1.3714	1.4613	49
12	1.3092	1.5493	1.3290	1.5182	1.3499	1.4887	1.3718	1.4608	48
13	1.3096	1.5487	1.3294	1.5177	1.3502	1.4882	1.3722	1.4604	47
14	1.3099	1.5482	1.3297	1.5171	1.3506	1.4877	1.3725	1.4599	46
15	1.3102	1.5477	1.3301	1.5166	1.3509	1.4873	1.3729	1.4595	45
16	1.3105	1.5471	1.3304	1.5161	1.3513	1.4868	1.3733	1.4590	44
17	1.3109	1.5466	1.3307	1.5156	1.3517	1.4863	1.3737	1.4586	43
18	1.3112	1.5461	1.3311	1.5151	1.3520	1.4858	1.3740	1.4581	42
19	1.3115	1.5456	1.3314	1.5146	1.3524	1.4854	1.3744	1.4577	41
20	1.3118	1.5450	1.3318	1.5141	1.3527	1.4849	1.3748	1.4572	40
21	1.3121	1.5445	1.3321	1.5136	1.3531	1.4844	1.3752	1.4568	39
22	1.3125	1.5440	1.3324	1.5131	1.3534	1.4839	1.3756	1.4563	38
23	1.3128	1.5434	1.3328	1.5126	1.3538	1.4835	1.3759	1.4559	37
24	1.3131	1.5429	1.3331	1.5121	1.3542	1.4830	1.3763	1.4554	36
25	1.3134	1.5424	1.3335	1.5116	1.3545	1.4825	1.3767	1.4550	35
26	1.3138	1.5419	1.3338	1.5111	1.3549	1.4821	1.3771	1.4545	34
27	1.3141	1.5413	1.3342	1.5106	1.3552	1.4816	1.3774	1.4541	33
28	1.3144	1.5408	1.3345	1.5101	1.3556	1.4811	1.3778	1.4536	32
29	1.3148	1.5403	1.3348	1.5096	1.3560	1.4806	1.3782	1.4532	31
30	1.3151	1.5398	1.3352	1.5092	1.3563	1.4802	1.3786	1.4527	30
31	1.3154	1.5392	1.3355	1.5087	1.3567	1.4797	1.3790	1.4523	29
32	1.3157	1.5387	1.3359	1.5082	1.3571	1.4792	1.3794	1.4518	28
33	1.3161	1.5382	1.3362	1.5077	1.3574	1.4788	1.3797	1.4514	27
34	1.3164	1.5377	1.3366	1.5072	1.3578	1.4783	1.3801	1.4510	26
35	1.3167	1.5371	1.3369	1.5067	1.3581	1.4778	1.3805	1.4505	25
36	1.3170	1.5366	1.3372	1.5062	1.3585	1.4774	1.3809	1.4501	24
37	1.3174	1.5361	1.3376	1.5057	1.3589	1.4769	1.3813	1.4496	23
38	1.3177	1.5356	1.3379	1.5052	1.3592	1.4764	1.3816	1.4492	22
39	1.3180	1.5351	1.3383	1.5047	1.3596	1.4760	1.3820	1.4487	21
40	1.3184	1.5345	1.3386	1.5042	1.3600	1.4755	1.3824	1.4483	20
41	1.3187	1.5340	1.3390	1.5037	1.3603	1.4750	1.3828	1.4479	19
42	1.3190	1.5335	1.3393	1.5032	1.3607	1.4746	1.3832	1.4474	18
43	1.3193	1.5330	1.3397	1.5027	1.3611	1.4741	1.3836	1.4470	17
44	1.3197	1.5325	1.3400	1.5022	1.3614	1.4736	1.3839	1.4465	16
45	1.3200	1.5319	1.3404	1.5018	1.3618	1.4732	1.3843	1.4461	15
46	1.3203	1.5314	1.3407	1.5013	1.3622	1.4727	1.3847	1.4457	14
47	1.3207	1.5309	1.3411	1.5008	1.3625	1.4723	1.3851	1.4452	13
48	1.3210	1.5304	1.3414	1.5003	1.3629	1.4718	1.3855	1.4448	12
49	1.3213	1.5299	1.3418	1.4998	1.3633	1.4713	1.3859	1.4443	11
50	1.3217	1.5294	1.3421	1.4993	1.3636	1.4709	1.3863	1.4439	10
51	1.3220	1.5289	1.3425	1.4988	1.3640	1.4704	1.3867	1.4435	9
52	1.3223	1.5283	1.3428	1.4983	1.3644	1.4699	1.3870	1.4430	8
53	1.3227	1.5278	1.3432	1.4979	1.3647	1.4695	1.3874	1.4426	7
54	1.3230	1.5273	1.3435	1.4974	1.3651	1.4690	1.3878	1.4422	6
55	1.3233	1.5268	1.3439	1.4969	1.3655	1.4686	1.3882	1.4417	5
56	1.3237	1.5263	1.3442	1.4964	1.3658	1.4681	1.3886	1.4413	4
57	1.3240	1.5258	1.3446	1.4059	1.3662	1.4676	1.3890	1.4408	3
58	1.3243	1.5253	1.3449	1.4954	1.3666	1.4672	1.3894	1.4404	2
59	1.3247	1.5248	1.3453	1.4949	1.3669	1.4667	1.3898	1.4400	1
60	1.3250	1.5242	1.3456	1.4945	1.3673	1.4663	1.3902	1.4395	0
′	49° Co-sec.	49° Sec.	48° Co-sec.	48° Sec.	47° Co-sec.	47° Sec.	46° Co-sec.	46° Sec.	′

	44°				44°				44°		
′	Sec.	Co-sec.	′	′	Sec.	Co-sec.	′	′	Sec	Co-sec.	′
0	1.3902	1.4395	60	21	1.3984	1.4305	39	41	1.4065	1.4221	19
1	1.3905	1.4391	59	22	1.3988	1.4301	38	42	1.4069	1.4217	18
2	1.3909	1.4387	58	23	1.3992	1.4297	37	43	1.4073	1.4212	17
3	1.3913	1.4382	57	24	1.3996	1.4292	36	44	1.4077	1.4208	16
4	1.3917	1.4378	56	25	1.4000	1.4288	35	45	1.4081	1.4204	15
5	1.3921	1.4374	55	26	1.4004	1.4284	34	46	1.4085	1.4200	14
6	1.3925	1.4370	54	27	1.4008	1.4280	33	47	1.4089	1.4196	13
7	1.3929	1.4365	53	28	1.4012	1.4276	32	48	1.4093	1.4192	12
8	1.3933	1.4361	52	29	1.4016	1.4271	31	49	1.4097	1.4188	11
9	1.3937	1.4357	51	30	1.4020	1.4267	30	50	1.4101	1.4183	10
10	1.3941	1.4352	50	31	1.4024	1.4263	29	51	1.4105	1.4179	9
11	1.3945	1.4348	49	32	1.4028	1.4259	28	52	1.4109	1.4175	8
12	1.3949	1.4344	48	33	1.4032	1.4254	27	53	1.4113	1.4171	7
13	1.3953	1.4339	47	34	1.4036	1.4250	26	54	1.4117	1.4167	6
14	1.3957	1.4335	46	35	1.4040	1.4246	25	55	1.4122	1.4163	5
15	1.3960	1.4331	45	36	1.4044	1.4242	24	56	1.4126	1.4159	4
16	1.3964	1.4327	44	37	1.4048	1.4238	23	57	1.4130	1.4154	3
17	1.3968	1.4322	43	38	1.4052	1.4233	22	58	1.4134	1.4150	2
18	1.3972	1.4318	42	39	1.4056	1.4229	21	59	1.4138	1.4146	1
19	1.3976	1.4314	41	40	1.4060	1.4225	20	60	1.4142	1.4142	0
20	1.3980	1.4310	40								
′	Co-sec.	Sec.	′	′	Co-sec.	Sec.	′	′	Co-sec.	Sec	′
	45°				45°				45°		

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